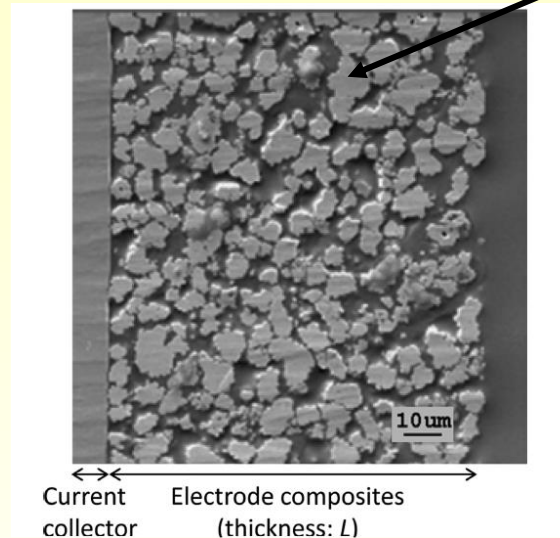
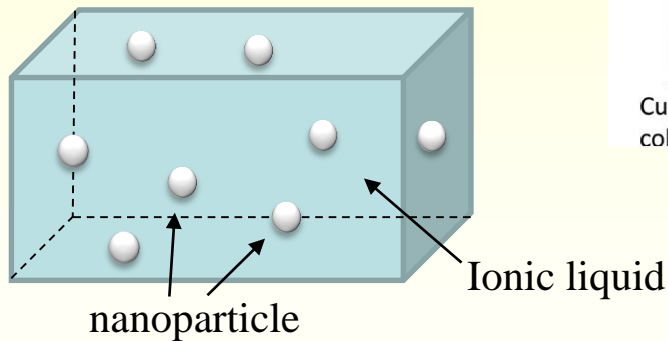
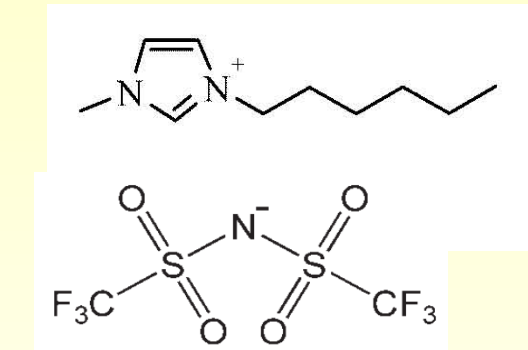


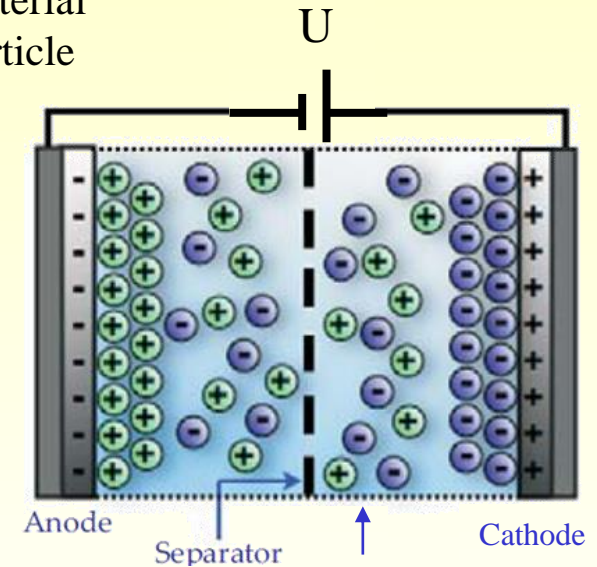
# Ionic Materials:

## Information from Impedance and Capacitance Spectra



Liquid electrolyte  
floods pore space

Active  
material  
particle

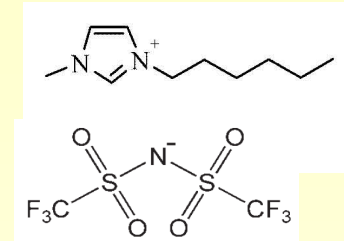


Bernhard Roling

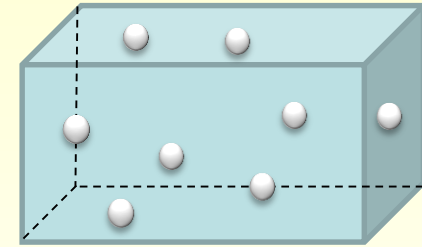
Department of Chemistry, University of Marburg

# Outline

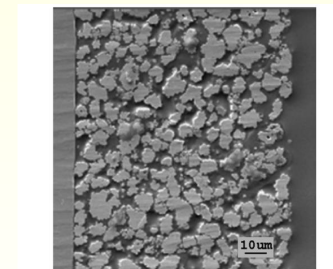
## 1. Ion Transport in Homogeneous Materials



## 2. Ion Transport in Heterogeneous Materials

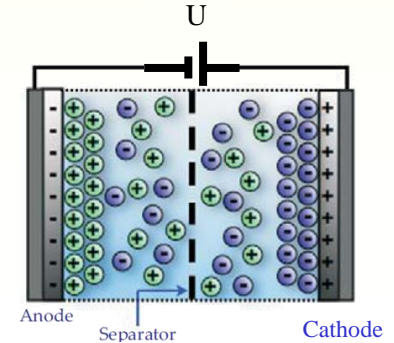


## 3. Ion Transport in Porous Battery Electrodes

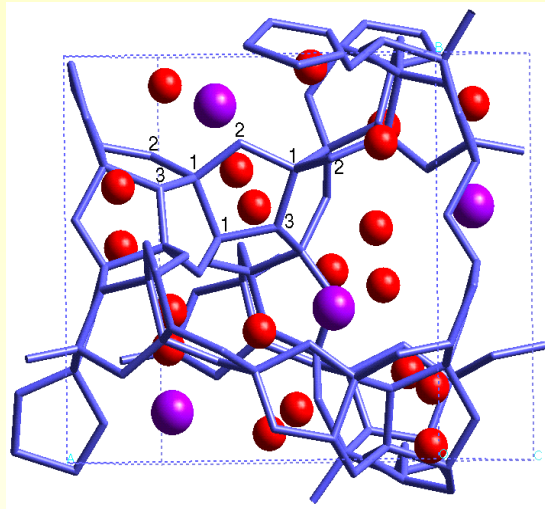


Current collector ← Electrode composites (thickness:  $L$ ) →

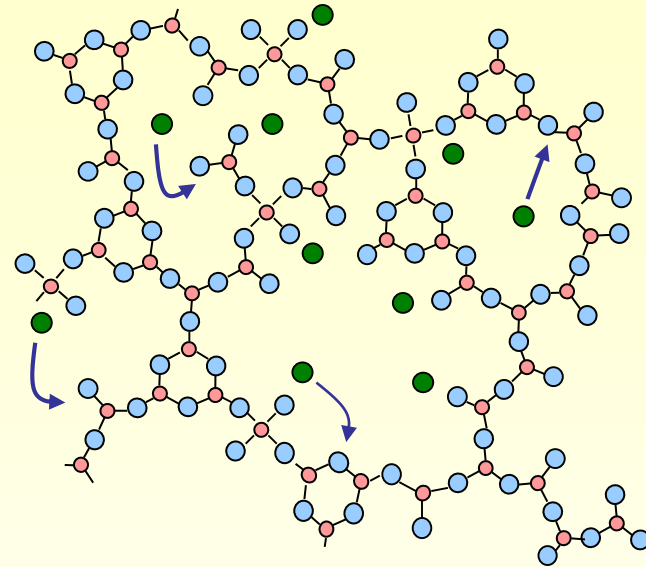
## 4. Double Layer Formation at Electrode Surfaces



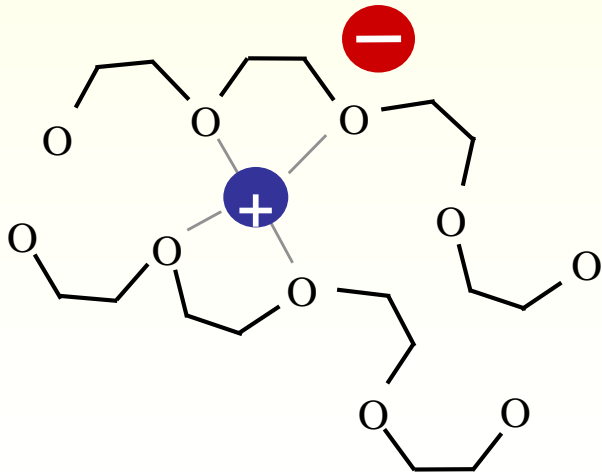
# 1. Ion Transport in Homogeneous Materials



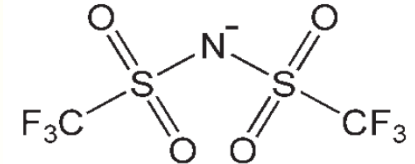
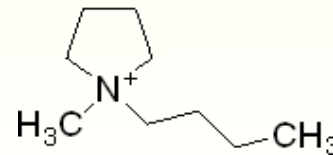
Single crystals ( $\text{RbAg}_4\text{I}_5$ )



Inorganic glasses

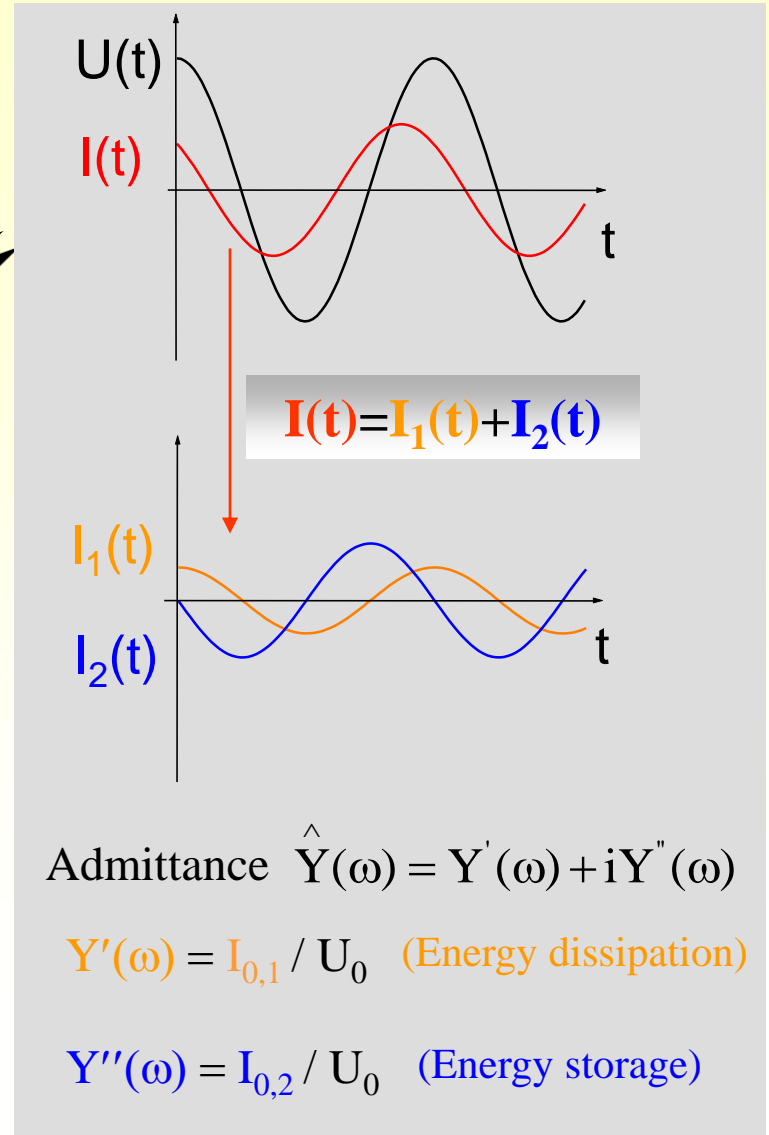
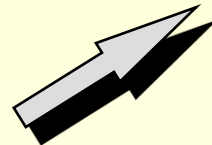
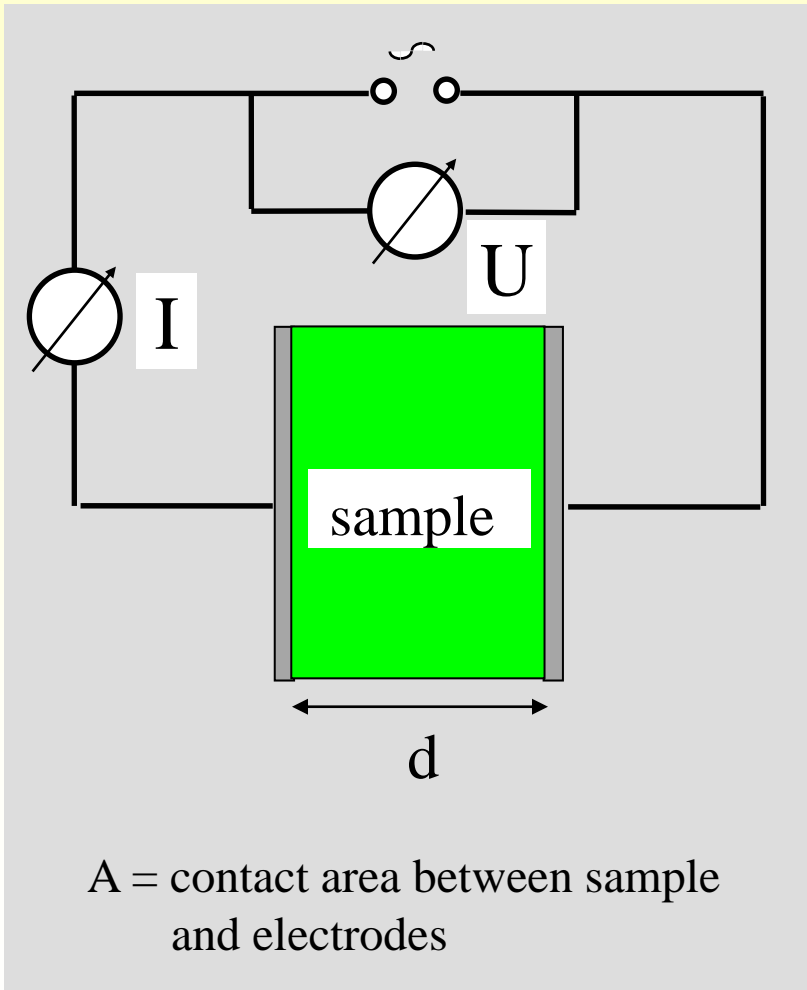


Amorphous polymers (PEO + Salt)



Ionic liquids

# Impedance Spectroscopy on Ion Conductor between Ion-Blocking Electrodes

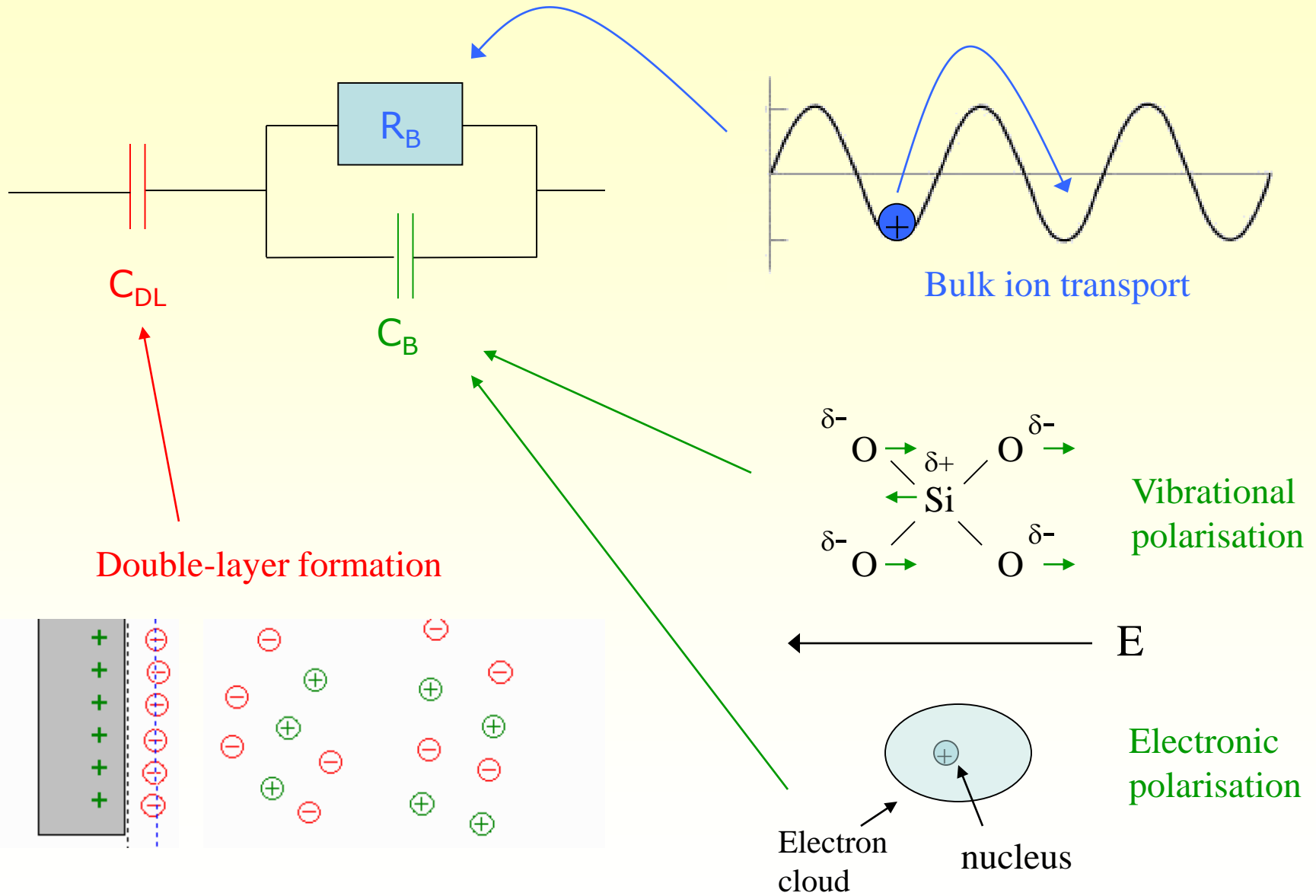


## Other quantities

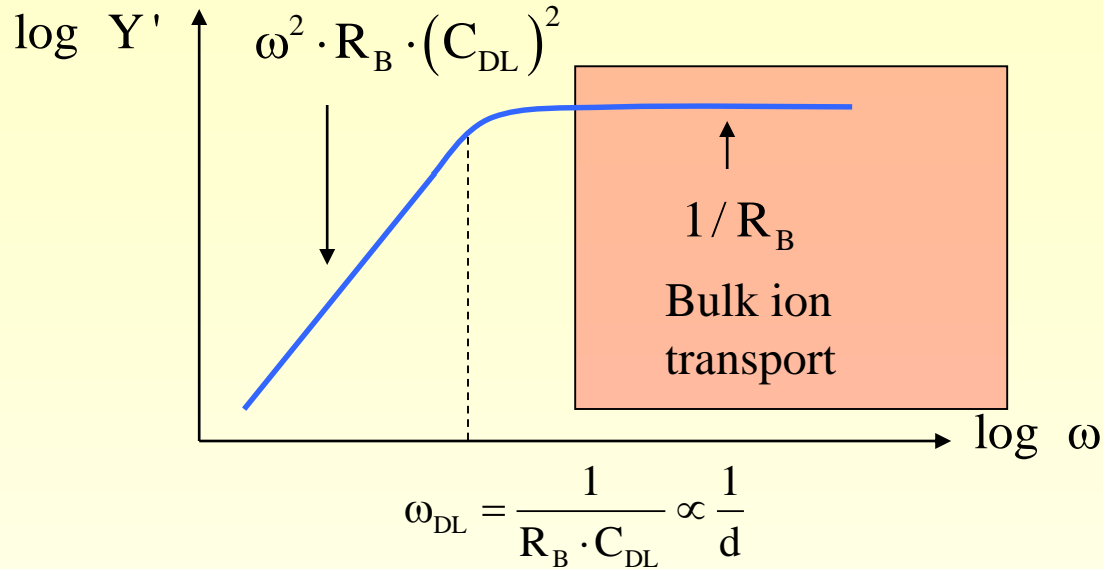
Impedance  $\hat{Z}(\omega) = \frac{1}{\hat{Y}(\omega)} = Z'(\omega) + i Z''(\omega)$

Capacitance  $\hat{C}(\omega) = \frac{\hat{Y}(\omega)}{i\omega} = \frac{1}{i\omega \hat{Z}(\omega)} = C'(\omega) + i C''(\omega)$

# Simplest equivalent circuit for *homogeneous* ion conductor between ion-blocking electrodes



## Real part of admittance

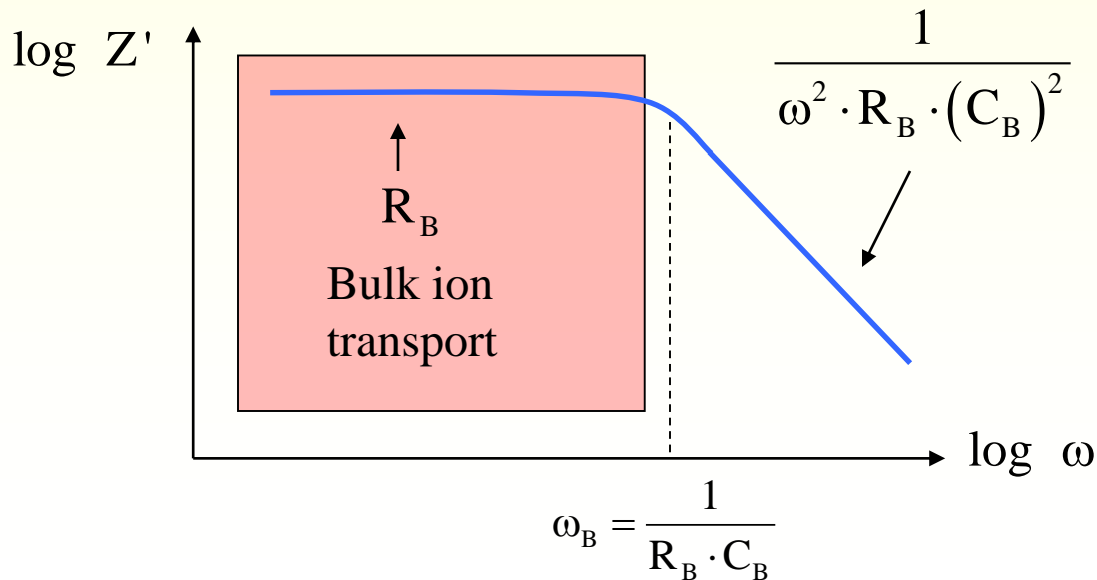


- Bulk ionic dc conductivity

$$\sigma_{dc} = \frac{1}{R_B} \cdot \frac{d}{A}$$

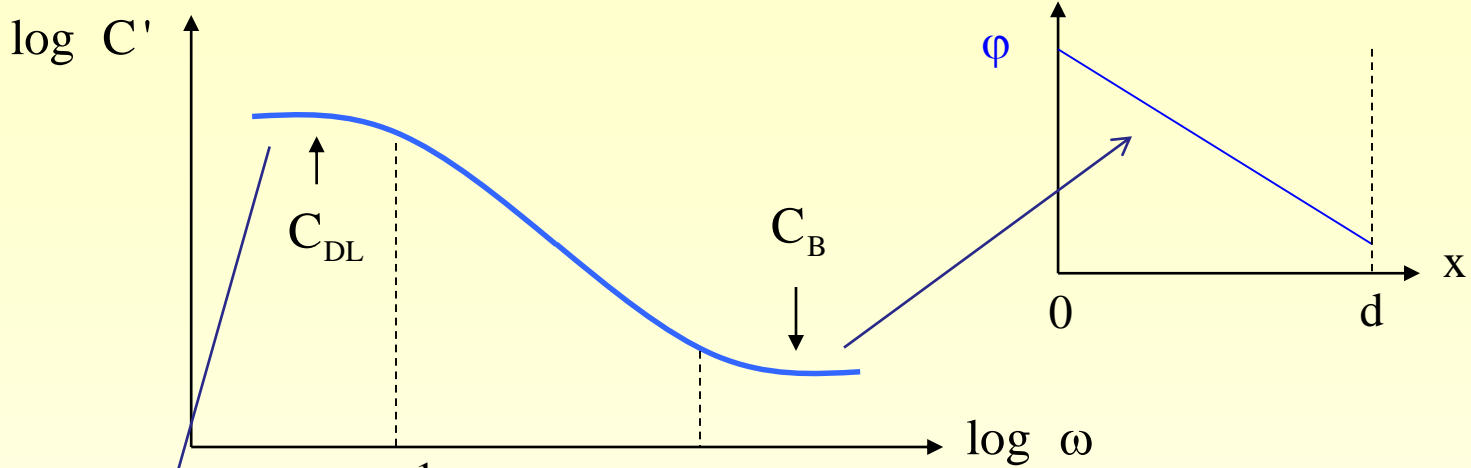
- No information about  $C_B$

## Real part of impedance



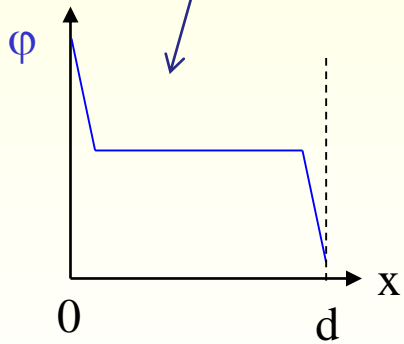
No information about  $C_{DL}$

# Real part of capacitance



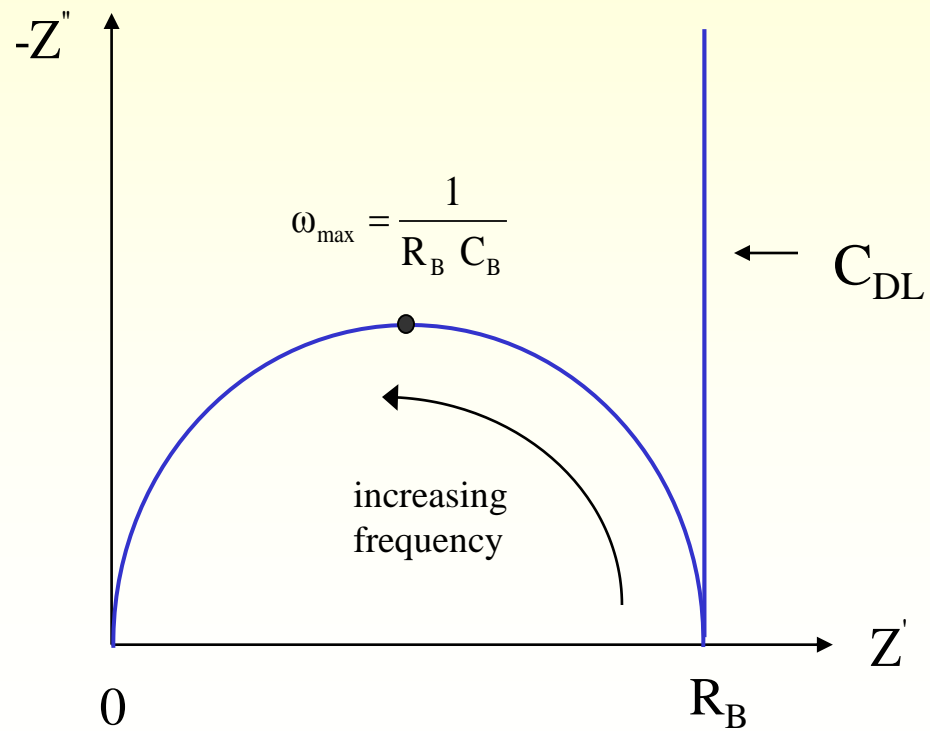
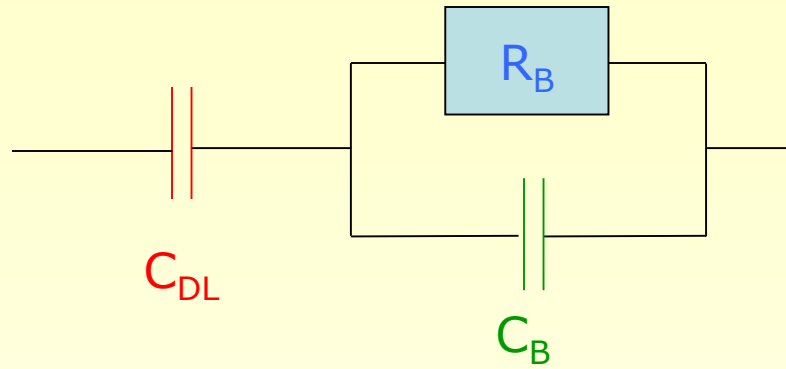
$$\omega_{DL} = \frac{1}{R_B \cdot C_{DL}} \propto \frac{1}{d}$$

$$\sqrt{\omega_B \cdot \omega_{DL}} = \sqrt{\frac{1}{(R_B \cdot C_B) \cdot (R_B \cdot C_{DL})}} \propto \frac{1}{\sqrt{d}}$$



$$\frac{C_{DL}}{C_B} \approx \frac{\text{Thickness of sample}}{\text{Thickness of double layer}} \approx \frac{1 \text{ mm}}{1 \text{ nm}} = 10^6$$

# Nyquist plot of impedance



## Rather common mistake in the analysis of impedance spectra

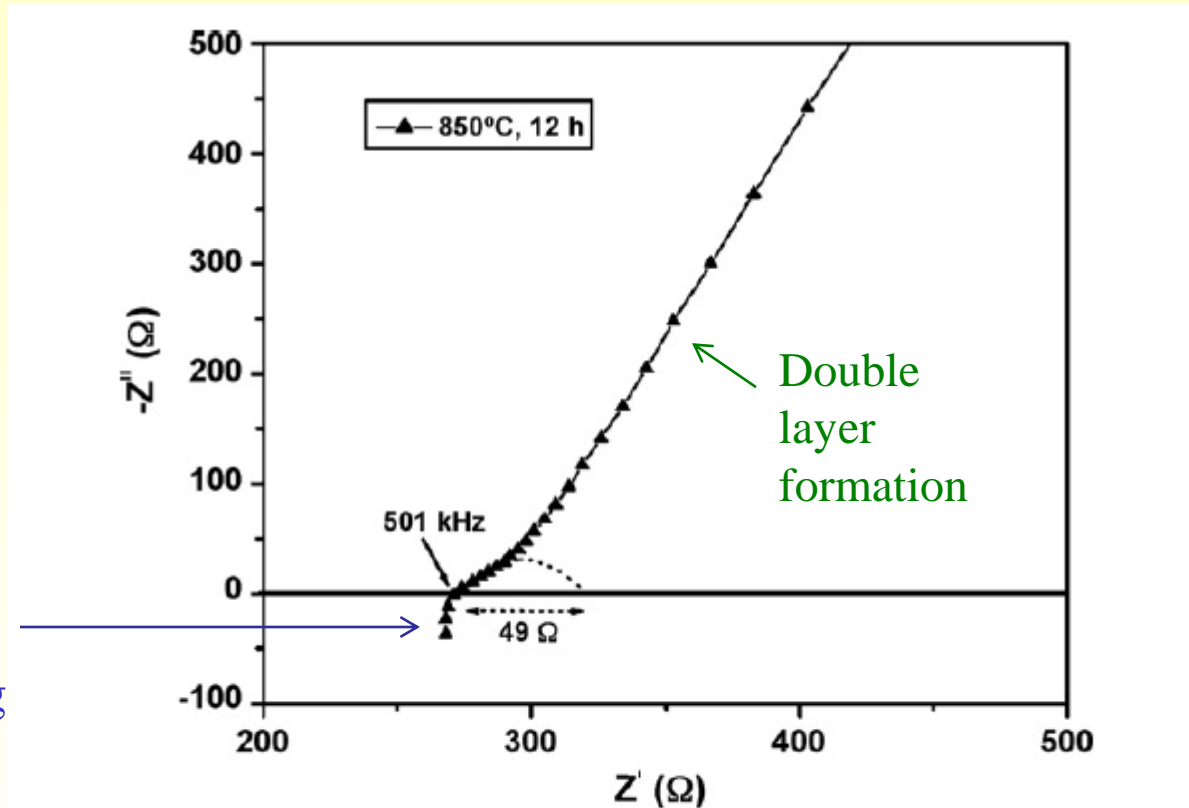
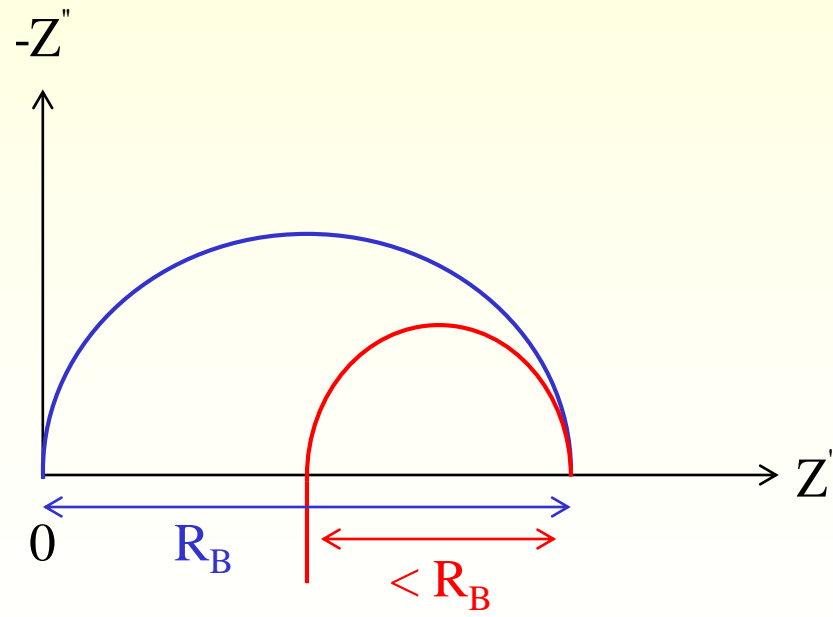
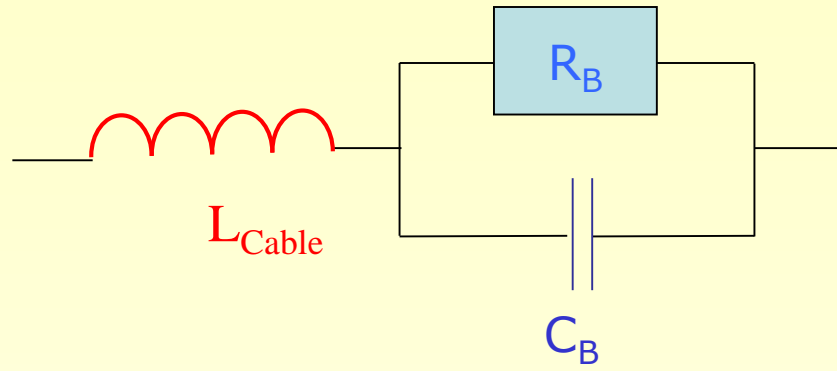
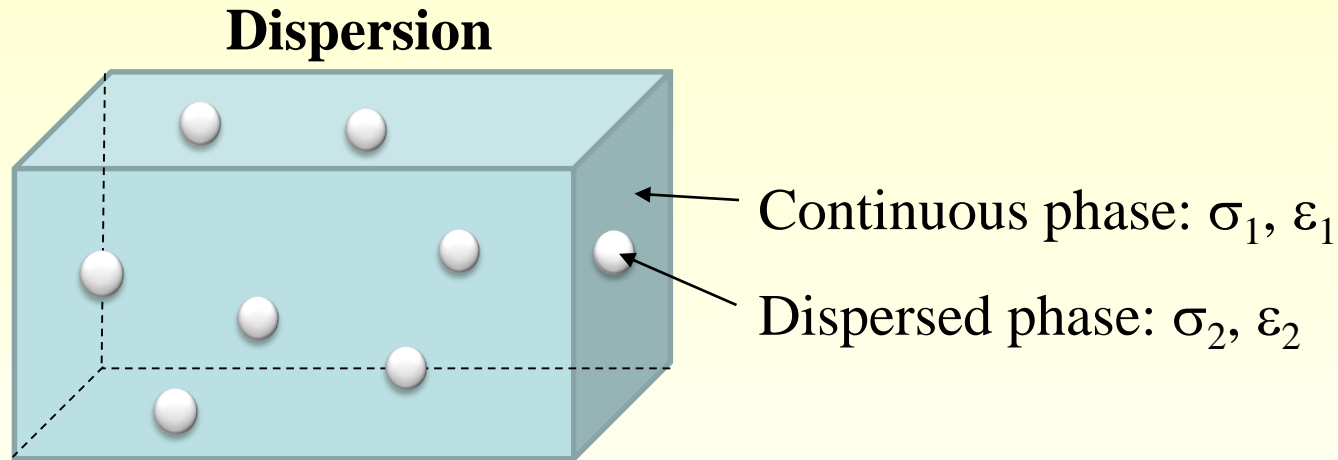


Fig. 4. Room temperature impedance spectra for LAGP glass sheet crystallized at 850°C for 12 h.

J. Thokchom, B. Kumar, J. Power Sources 185 (2008) 480.



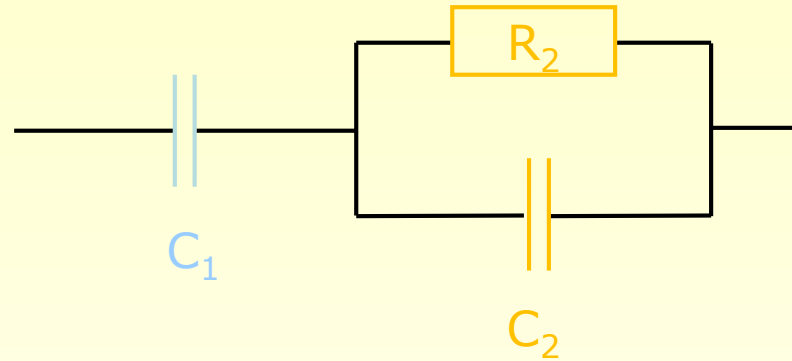
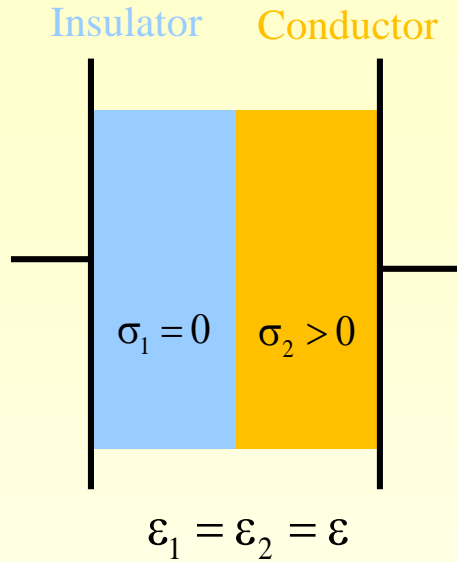
## 2. Ion Transport in Heterogeneous Materials



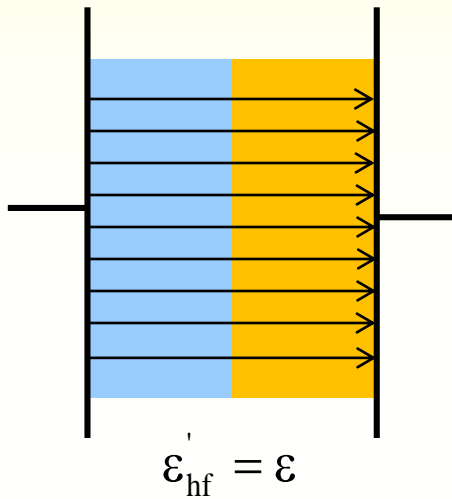
### Examples:

- Metallic particles in an insulating polymer matrix:  $\sigma_2 \gg \sigma_1$
- Insulating particles in a gel electrolyte:  $\sigma_1 \gg \sigma_2$

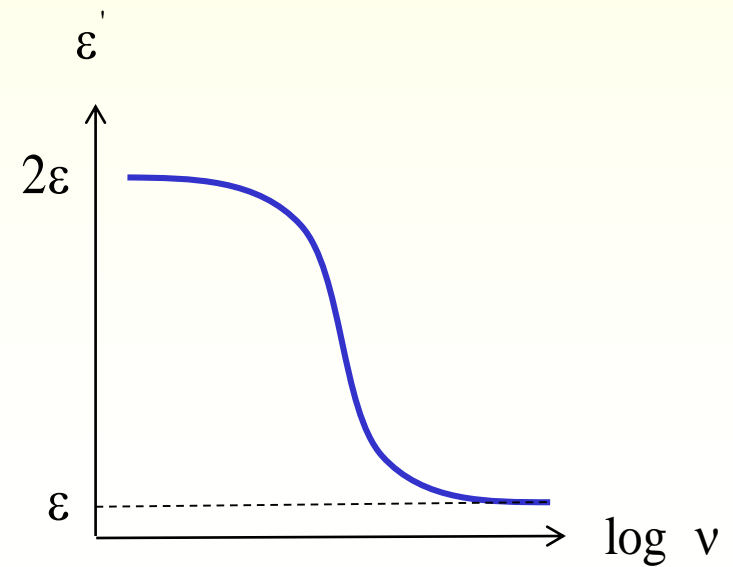
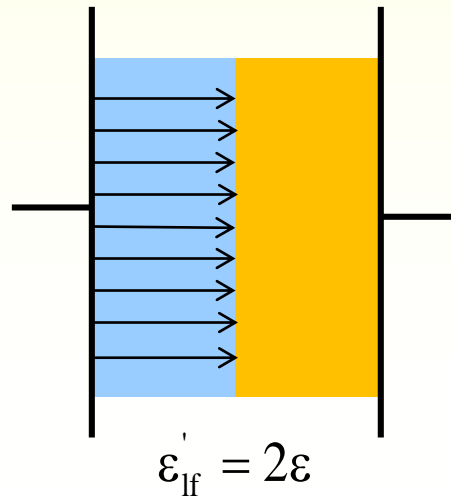
# Two serial phases



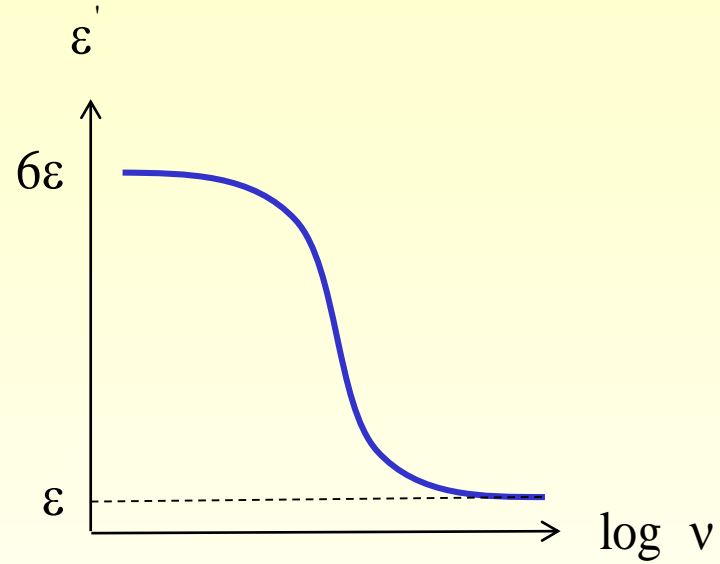
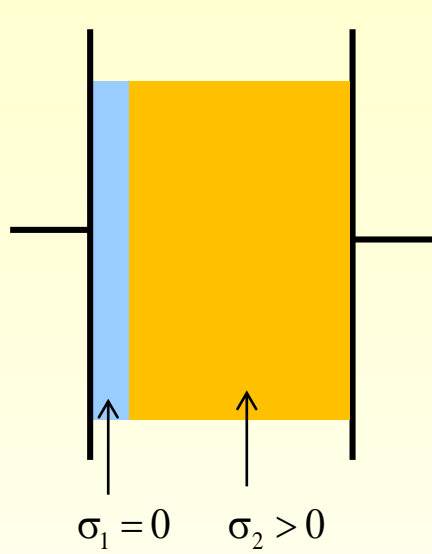
High frequencies:



Low frequencies:

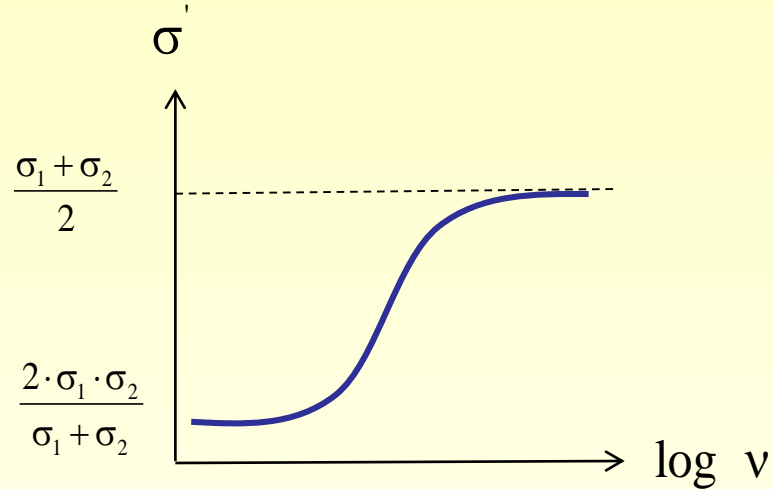
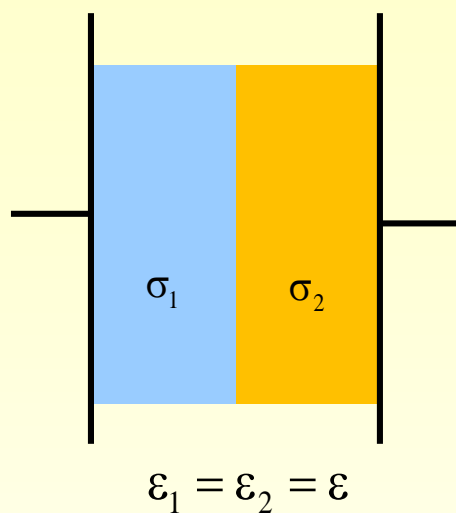


# Two serial phases



$$\frac{\epsilon''_{lf}}{\epsilon''_{hf}} = \frac{\text{Thickness of sample}}{\text{Thicknees of insulating phase}}$$

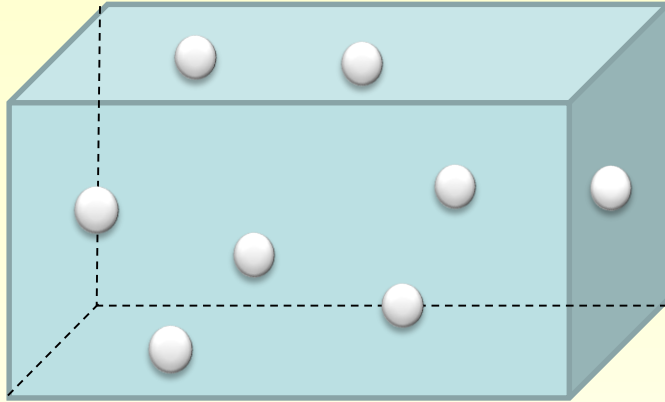
## Two serial phases



*High frequencies:* Averaging over the conductivity of different phases  
(Charge carriers move only locally)

*Low frequencies:* Averaging over the resistivity of different phases  
(Charge carriers move across the sample)

# Dispersions



Continuous phase:  $\sigma_1, \epsilon_1$

Dispersed phase:  $\sigma_2, \epsilon_2$

Volume fraction of dispersed phase:  $\phi$

## **Basic assumptions for Maxwell-Wagner-Bruggemann approach:**

- Dispersed particles are spheres.
- Small volume fraction  $\phi$  (diluted system)
- Random spatial distribution of dispersed particles  
(no interactions between dispersed particles)

# Maxwell-Wagner-Bruggemann equations

$$\hat{\sigma} = \sigma'_{\text{lf}} + \frac{(\sigma'_{\text{hf}} - \sigma'_{\text{lf}}) \cdot i\omega\tau}{1 + i\omega\tau} + i\omega\varepsilon_0\varepsilon'_{\text{hf}}$$

with

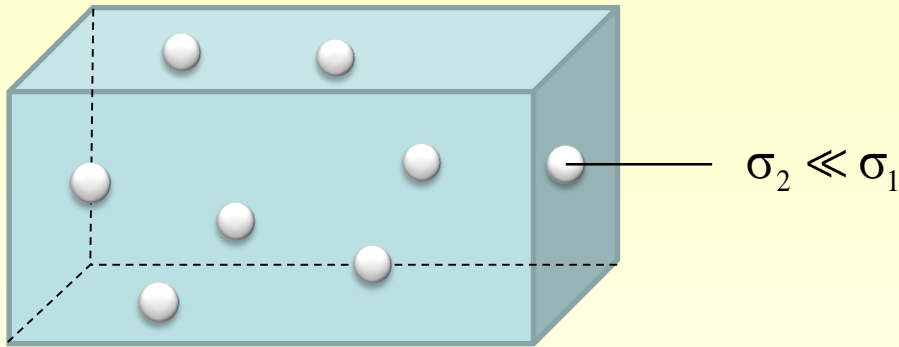
$$\sigma'_{\text{lf}} = \sigma_1 \frac{2\sigma_1 + \sigma_2 - 2\phi(\sigma_1 - \sigma_2)}{2\sigma_1 + \sigma_2 + \phi(\sigma_1 - \sigma_2)} \quad \text{Low-frequency conductivity}$$

$$\sigma'_{\text{hf}} - \sigma'_{\text{lf}} = \frac{9(\sigma_1\varepsilon_2 - \sigma_2\varepsilon_1)^2 \phi(1-\phi)}{(2\sigma_1 + \sigma_2 + \phi(\sigma_1 - \sigma_2))(2\varepsilon_1 + \varepsilon_2 + \phi(\varepsilon_1 - \varepsilon_2))^2} \quad \text{Difference between high-frequency and low-frequency conductivity}$$

$$\varepsilon'_{\text{hf}} = \varepsilon_1 \frac{2\varepsilon_1 + \varepsilon_2 - 2\phi(\varepsilon_1 - \varepsilon_2)}{2\varepsilon_1 + \varepsilon_2 + \phi(\varepsilon_1 - \varepsilon_2)} \quad \text{High-frequency permittivity}$$

$$\tau = \frac{\varepsilon_0(2\varepsilon_1 + \varepsilon_2 + \phi(\varepsilon_1 - \varepsilon_2))}{2\sigma_1 + \sigma_2 + \phi(\sigma_1 - \sigma_2)} \quad \text{Relaxation time}$$

## Limiting case: Dispersed insulator

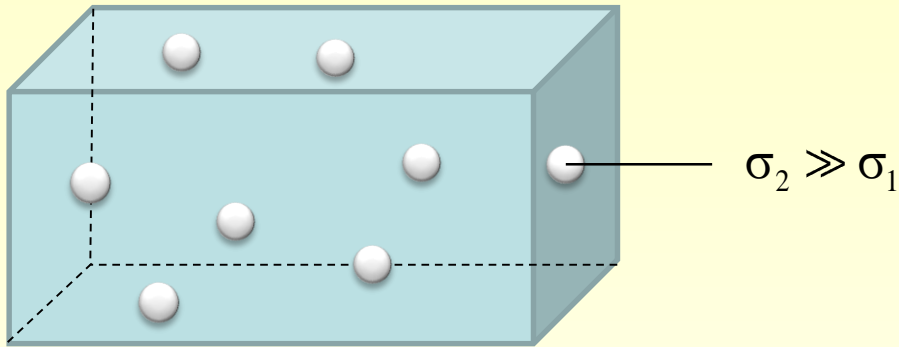


Low-frequency conductivity

$$\sigma'_{\text{lf}} = \sigma_1 \frac{2\sigma_1 + \sigma_2 - 2\phi(\sigma_1 - \sigma_2)}{2\sigma_1 + \sigma_2 + \phi(\sigma_1 - \sigma_2)} \approx \sigma_1 \left( 1 - \frac{3}{2}\phi \right)$$

The addition of 1% insulating particles results in a conductivity drop of about 1.5 %

## Limiting case: Dispersed conductor



Low-frequency conductivity

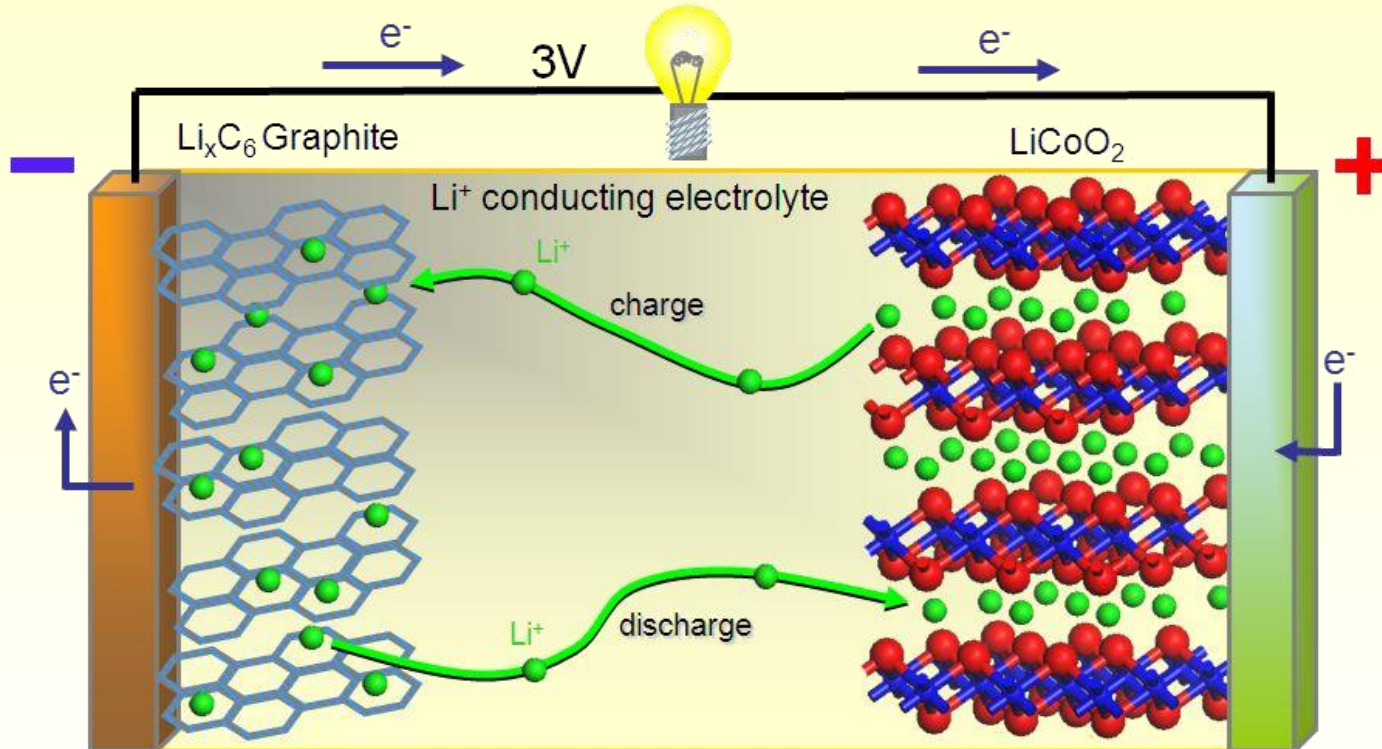
$$\sigma_{\text{lf}} = \sigma_1 \frac{2\sigma_1 + \sigma_2 - 2\phi(\sigma_1 - \sigma_2)}{2\sigma_1 + \sigma_2 + \phi(\sigma_1 - \sigma_2)} \approx \sigma_1 (1 + 3\phi)$$

The addition of 1% conducting particles results in a conductivity rise of about 3%

Note: Since the conducting particles do not form percolating pathways, the composite conductivity is governed by the conductivity of the insulating phase,  $\sigma_1$ .

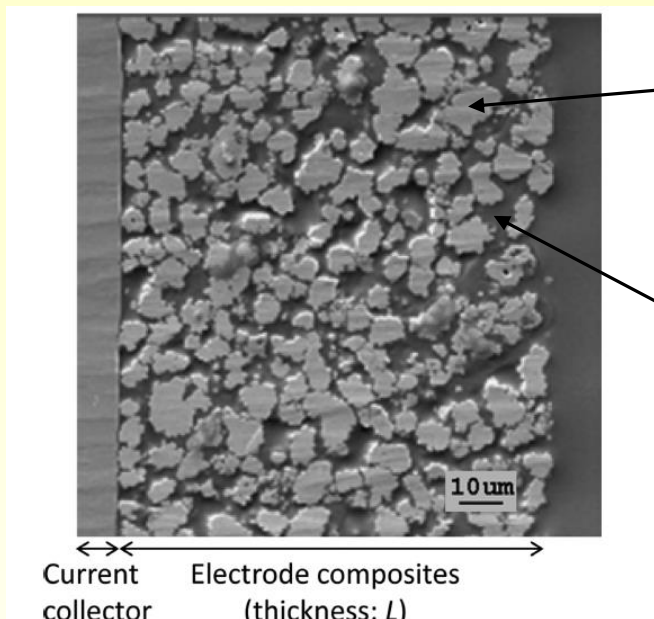
### 3. Ion Transport in Porous Battery Electrodes

#### Example: Li-ion battery



Gravimetric energy density:  
**150-200 Wh / kg**

# Composite electrodes

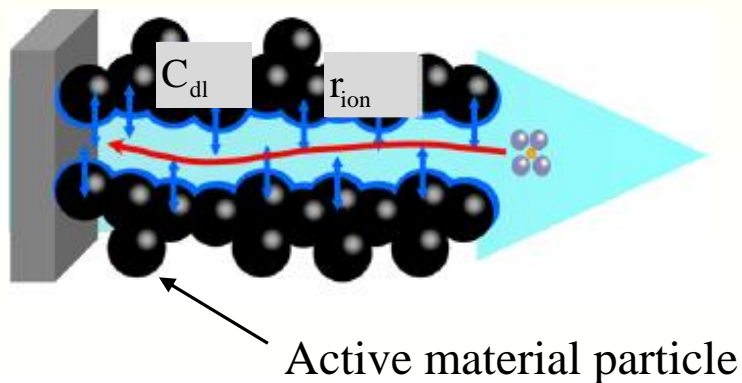


Active material particle (stores Li)

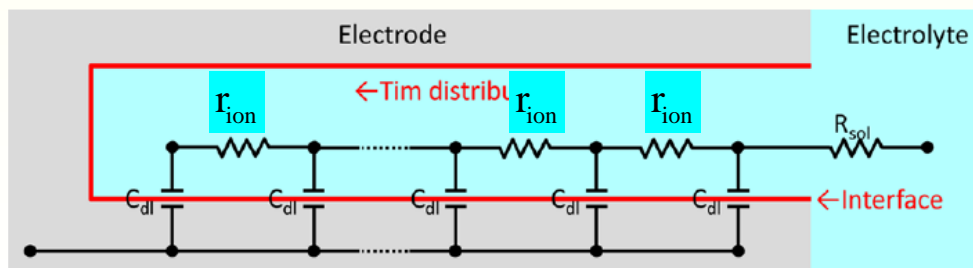
Pore space is flooded by the liquid electrolyte

Ogihara et al,  
*J. Phys. Chem.*  
**119** (2015) 4612

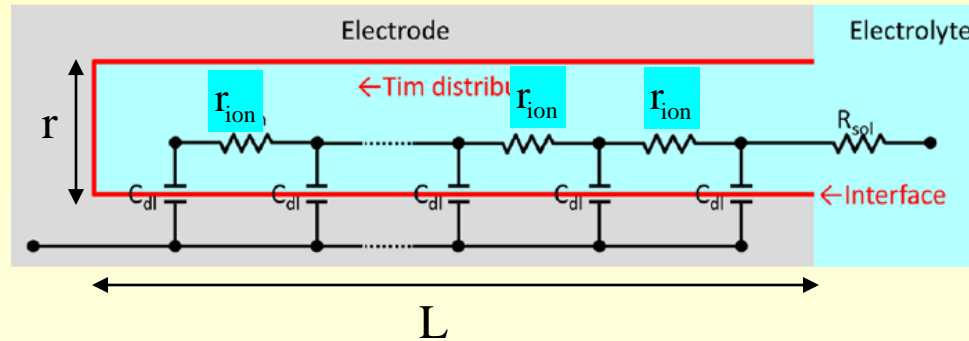
## Idealized model approach with cylindrical pores:



## Transmission line circuit



# Transmission line model



$$r_{ion} = \frac{1}{\sigma_{ion} \cdot 4\pi r^2} \quad \text{Electrolyte resistance per unit length } (\Omega / \text{m})$$

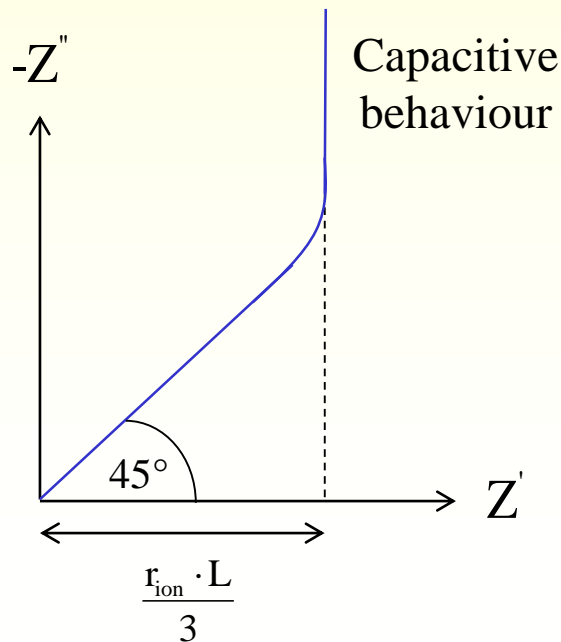
$$C_{dl} = \frac{C_{dl,pore}}{2\pi r L} \quad \text{Double layer capacitance per unit area } (\text{F} / \text{m}^2)$$

$$\hat{Z} = \sqrt{\frac{r_{ion}}{i\omega C_{dl} \cdot 2\pi r}} \cdot \coth\left(\sqrt{r_{ion} \cdot i\omega C_{dl} \cdot 2\pi r} \cdot L\right)$$

$$\hat{Z} = \sqrt{\frac{r_{\text{ion}}}{i\omega C_{\text{dl}} \cdot 2\pi r}} \cdot \coth\left(\sqrt{r_{\text{ion}} \cdot i\omega C_{\text{dl}} \cdot 2\pi r \cdot L}\right)$$

High frequencies:  $\coth\left(\sqrt{R_{\text{ion}} \cdot i\omega C_{\text{dl}} \cdot 2\pi r \cdot L}\right) \rightarrow 1$

$$\longrightarrow \hat{Z} \propto (i\omega)^{-1/2}$$



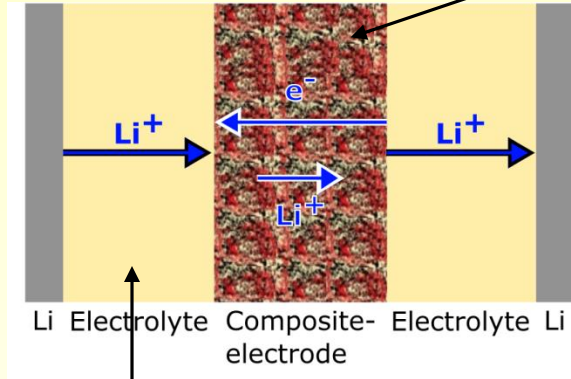
Low frequencies:  $\coth(x) \approx \frac{1}{x} + \frac{x}{3}$

$$\longrightarrow \hat{Z} = \frac{r_{\text{ion}} \cdot L}{3} - i \frac{1}{\omega C_{\text{dl}} \cdot 2\pi r L}$$

Ion transport resistance:  $R_{\text{ion}} = r_{\text{ion}} \cdot L$  ( $\Omega$ )

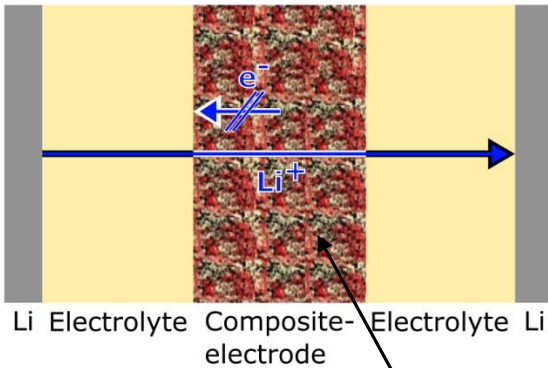
# Stationary Ion Transport Measurements

High frequencies:



$R_{\text{Electrolyte}}$

Low frequencies:

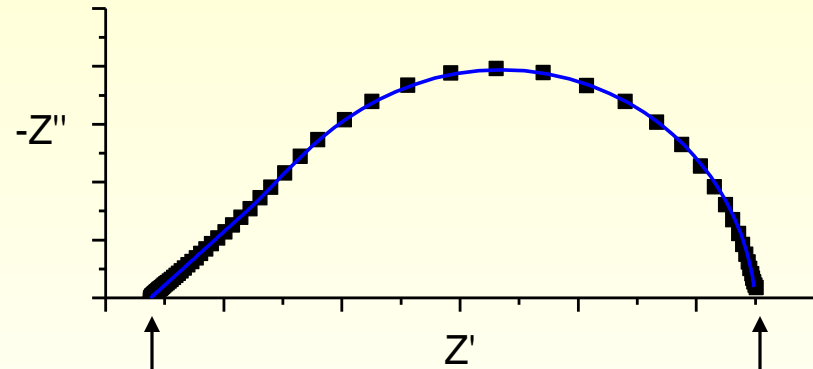


$R_{\text{composite}}(\omega \rightarrow 0) = R_{\text{ion}}$

$$R_{\text{composite}}(\omega \rightarrow \infty)$$

$$= \frac{R_{\text{ion}} \cdot R_{\text{electron}}}{R_{\text{ion}} + R_{\text{electron}}} \approx R_{\text{electron}}$$

Ion and electrons  
move in parallel



$$Z'(\omega \rightarrow \infty)$$

$$= 2 \cdot R_{\text{Electrolyte}} + R_{\text{electron}}$$

$$\approx 2 \cdot R_{\text{Electrolyte}}$$

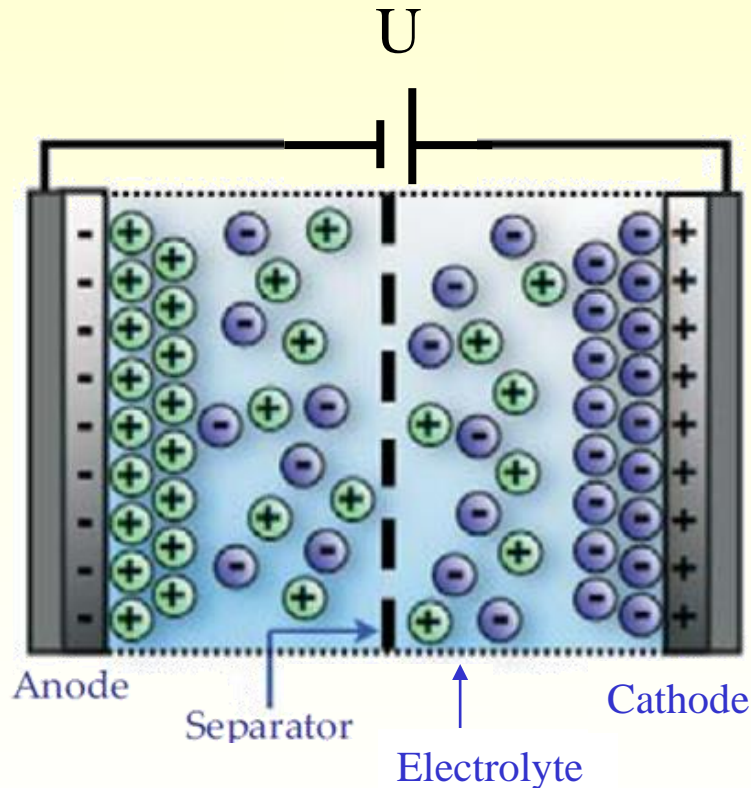
$$Z'(\omega \rightarrow 0)$$

$$= 2 \cdot R_{\text{Electrolyte}} + R_{\text{ion}}$$

$$R_{\text{ion}} = Z'(\omega \rightarrow \infty) - Z'(\omega \rightarrow 0)$$

## 4. Double Layer Formation at Electrode Surfaces

### Energy storage in electrochemical supercapacitors



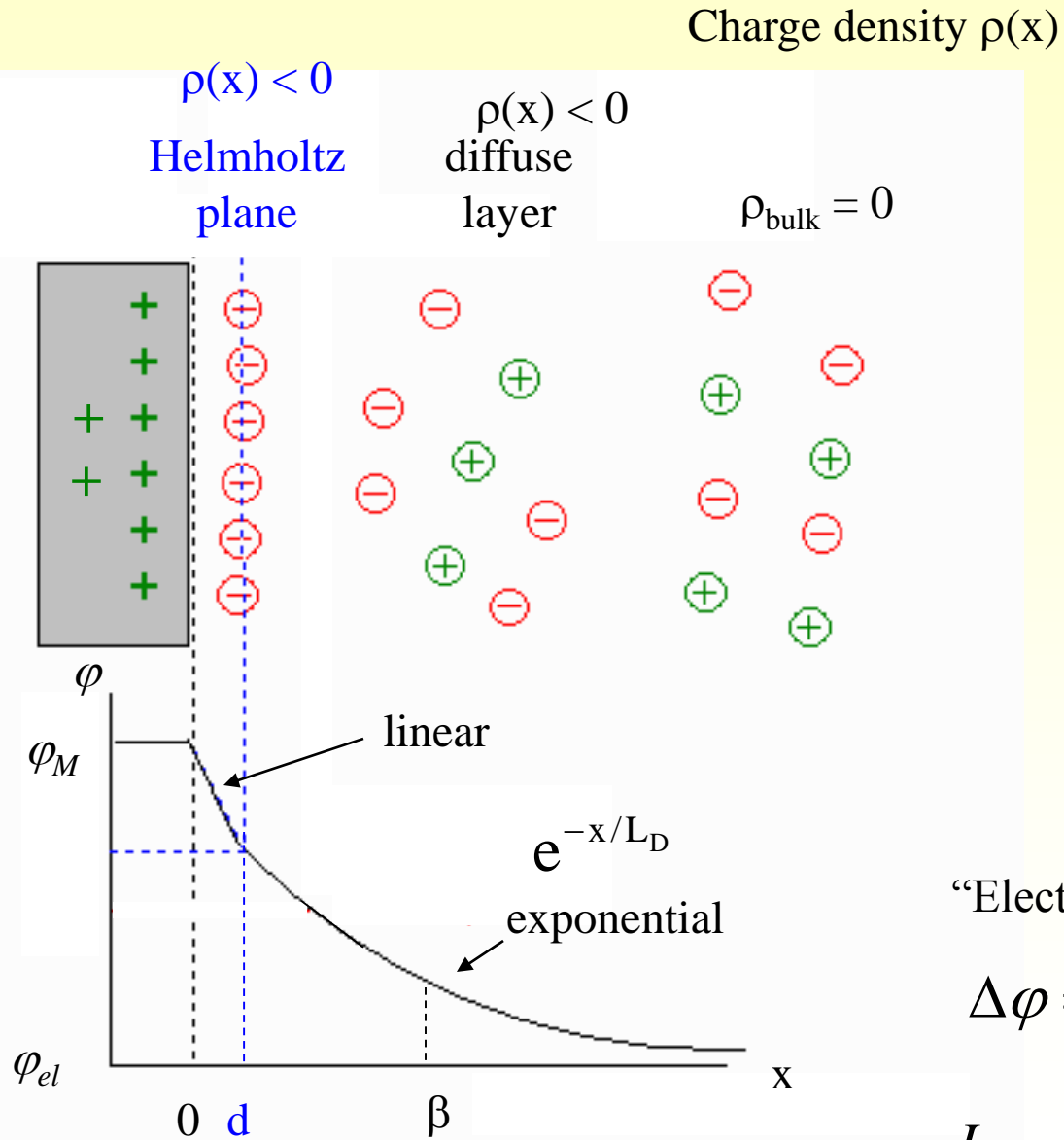
**Stored energy**

$$E = \frac{1}{2} \cdot C \cdot U^2$$

$C$  = double layer capacitance

$U$  = maximum voltage

# Classical Stern model for **diluted** electrolytes



“Electrode potential“

$$\Delta\varphi = \varphi_M - \varphi_{el}$$

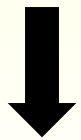
$L_D$  = Debye length

## Differential capacitance of the double layer

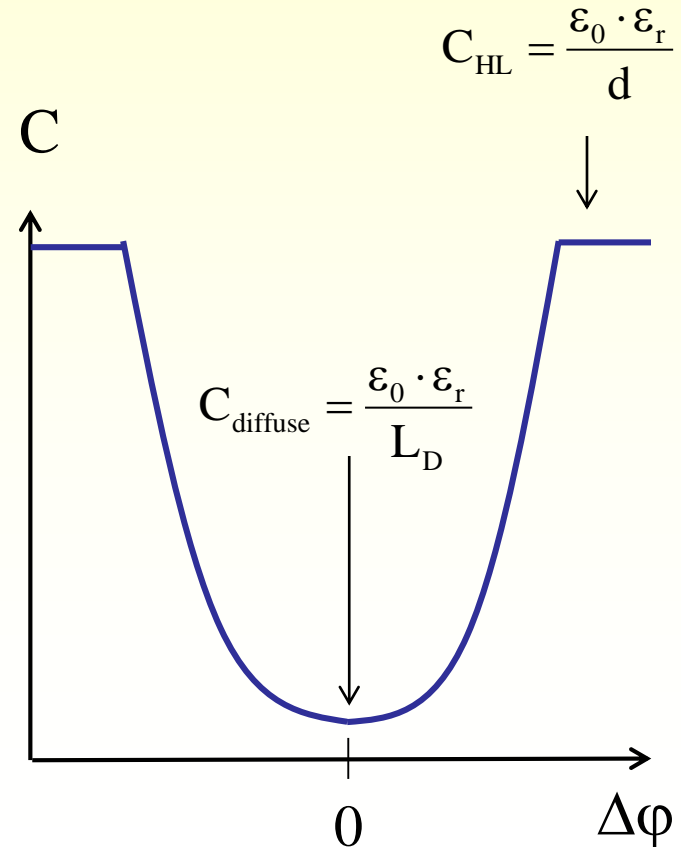
Differential capacitance per unit area  $C = \frac{d(Q_{\text{electrode}} / A)}{d(\Delta\varphi)}$

Helmholtz layer  $C_{\text{HL}} = \frac{\epsilon_0 \cdot \epsilon_r}{d}$

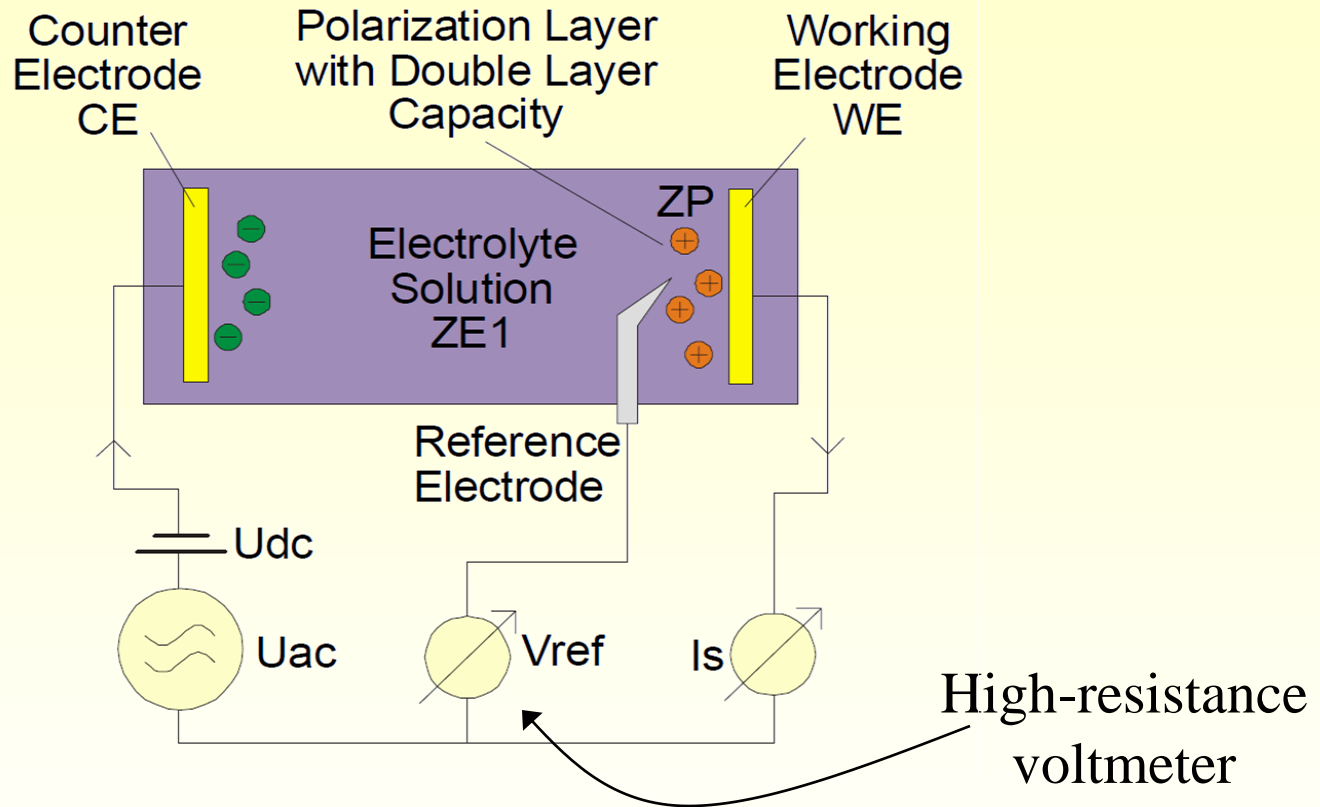
Diffuse layer  $C_{\text{diffuse}} = \frac{\epsilon_0 \cdot \epsilon_r}{L_D} \cdot \cosh\left(\frac{e \cdot \Delta\varphi}{2 \cdot kT}\right)$



$$\frac{1}{C} = \frac{1}{C_{\text{HL}}} + \frac{1}{C_{\text{diffuse}}}$$



# Experiment: Three-Electrode Configuration



Impedance of working electrode: 
$$\hat{Z}_{WE} = \frac{V_{WE} - V_{ref}}{I_s}$$

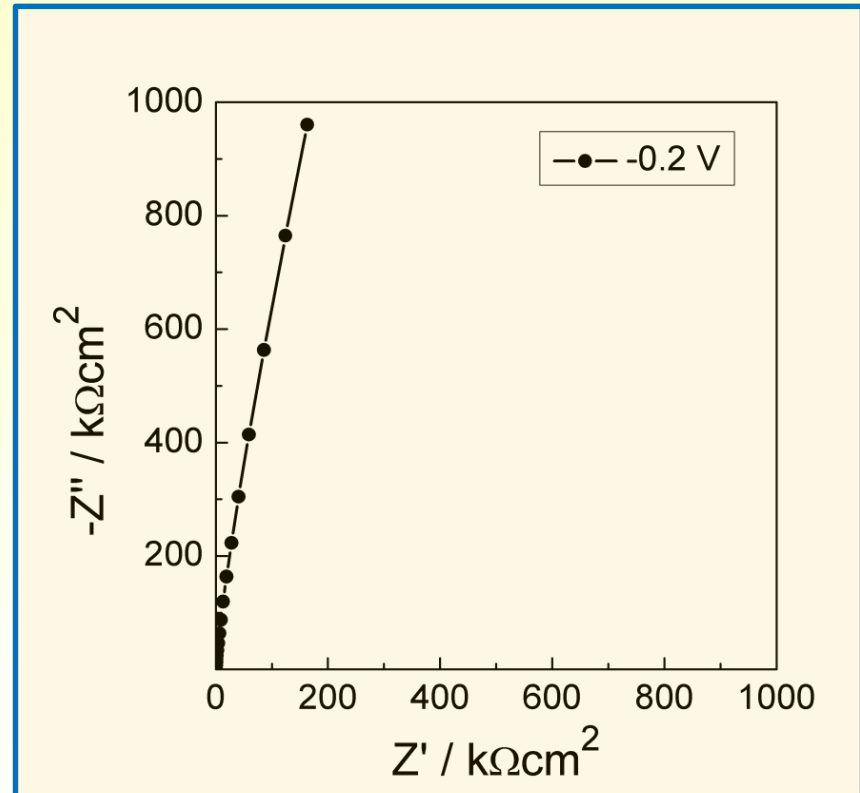
# Nyquist plot of impedance

Frequency range:

$\nu = 10 \text{ mHz} - 1 \text{ MHz}$

Ac potential

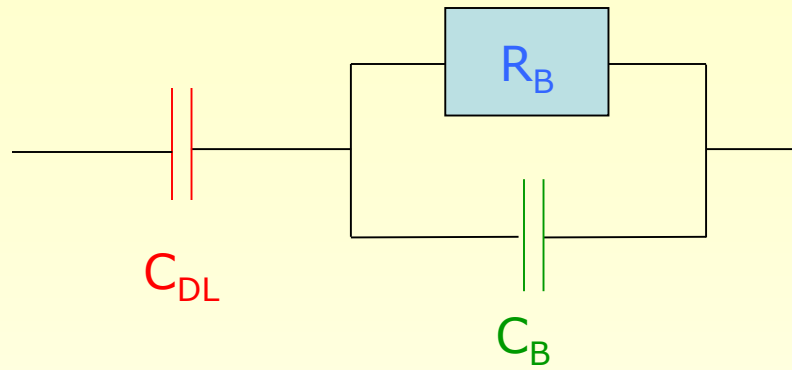
10 mV (rms)



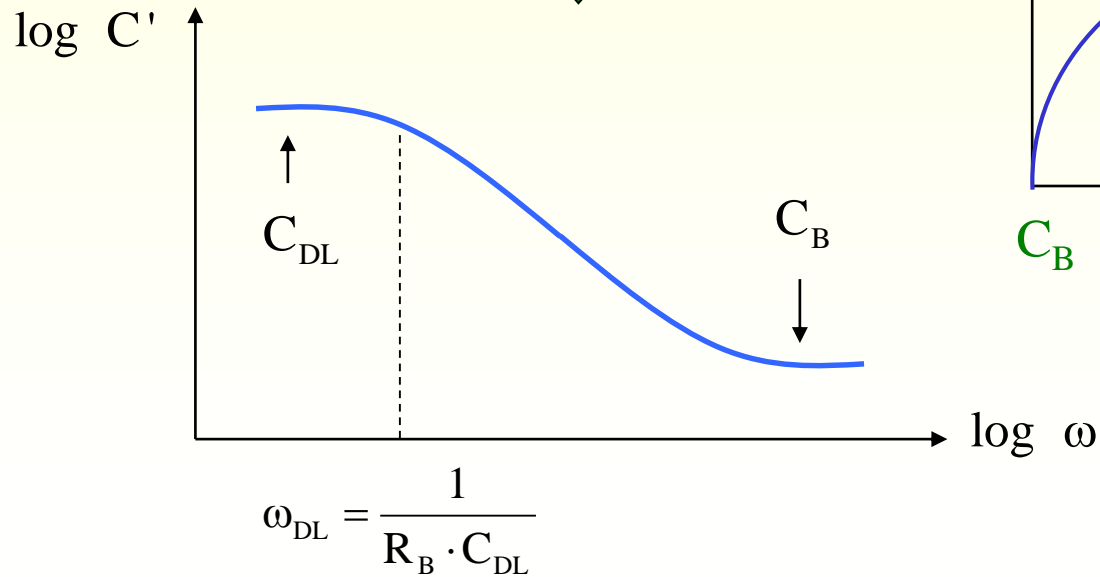
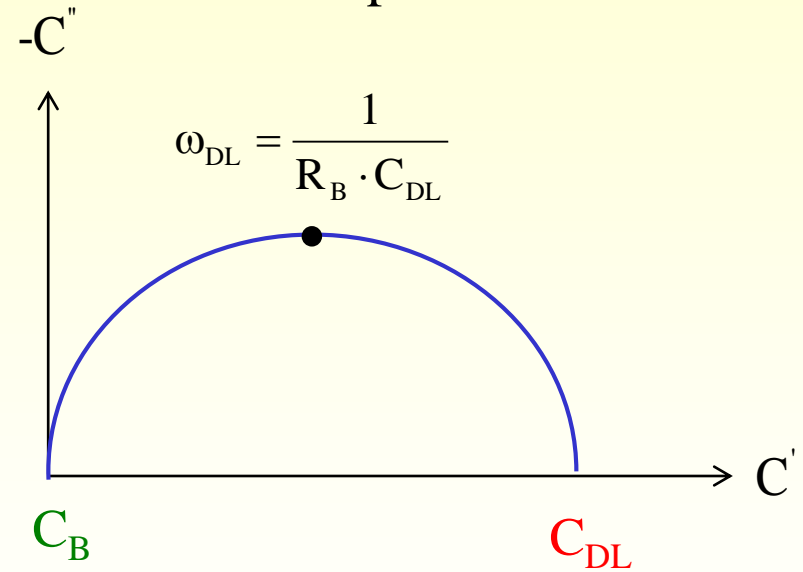
Non-ideal capacitance

M. Drüscher, B. Roling  
J. Phys. Chem. C  
115 (2011) 6802.

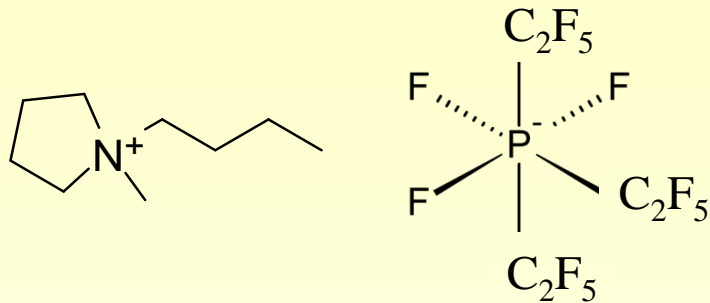
# Simplest equivalent circuit representation



Complex capacitance plane



# Complex capacitance plane of Pyr14-FAP at Au(111)



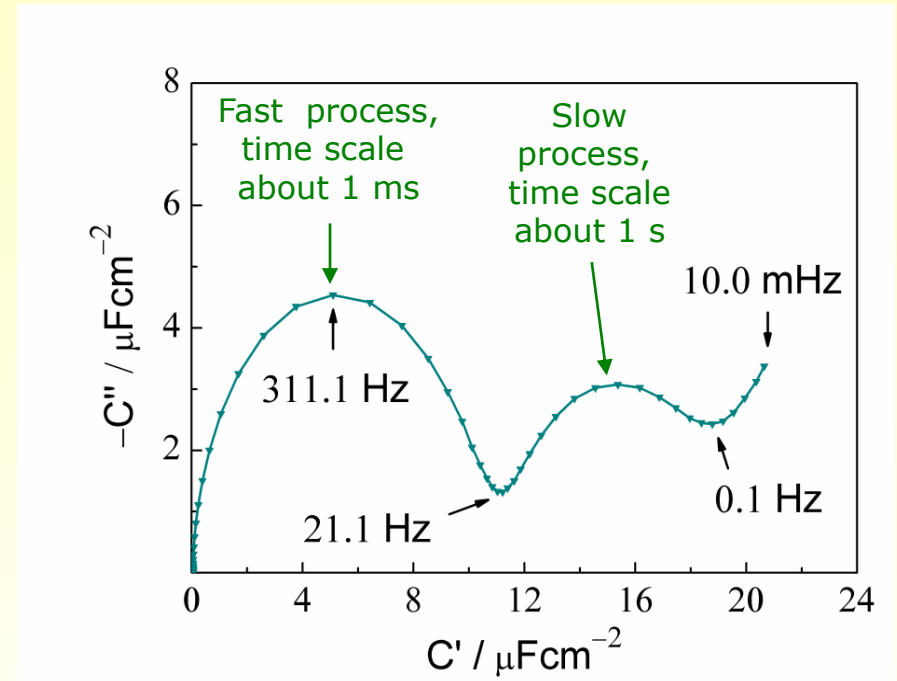
Ionic liquids:  $\tau_{DL} = R_{bulk} \cdot C_{DL} \approx 1 \text{ ms}$

Fit with empirical Cole-Cole relation:

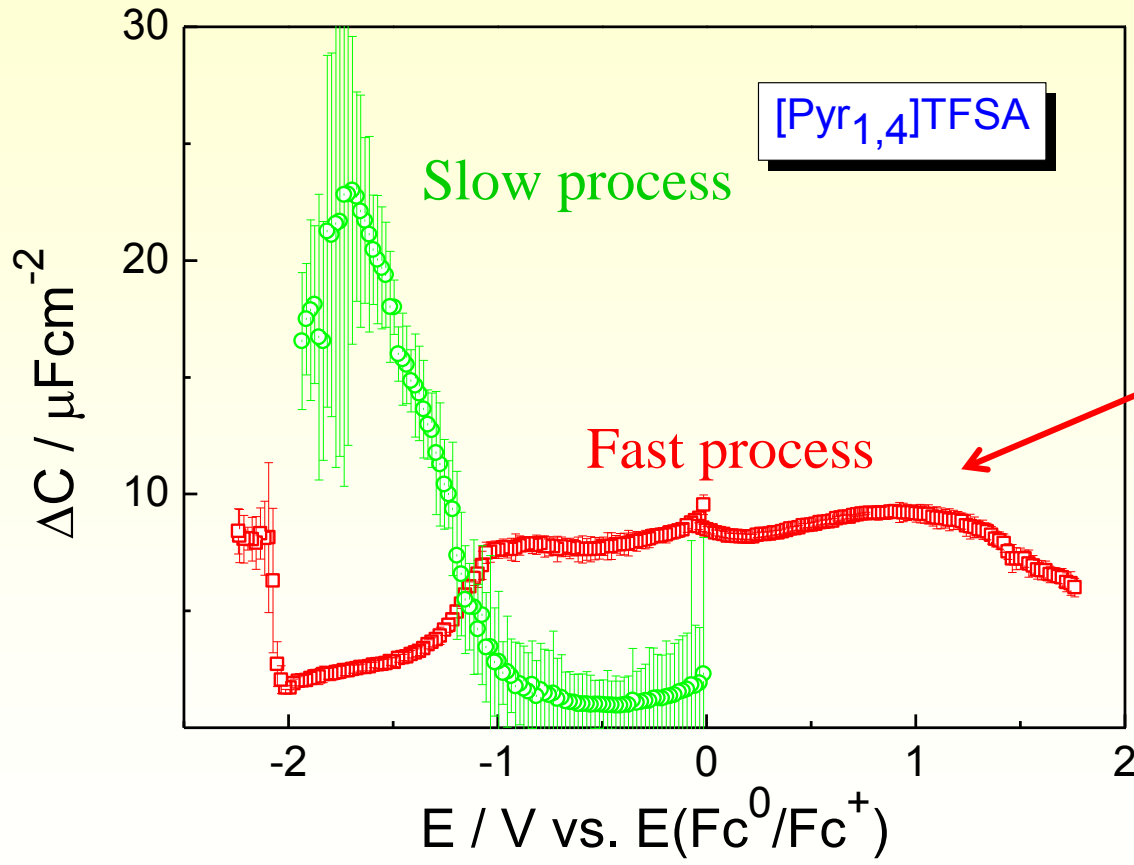
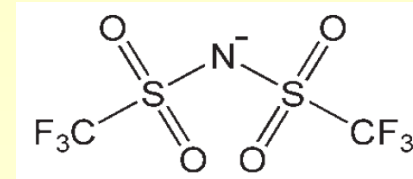
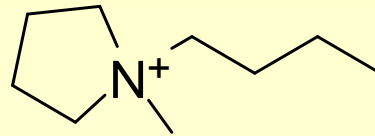
$$(\hat{C}(\nu) - C_{\infty}) = \sum_{i=1}^n \frac{\Delta C_i}{1 + (j2\pi\nu\tau_i)^{\alpha_i}}$$



Differential capacitance of both processes  
 $\Delta C_{fast}$  and  $\Delta C_{slow}$

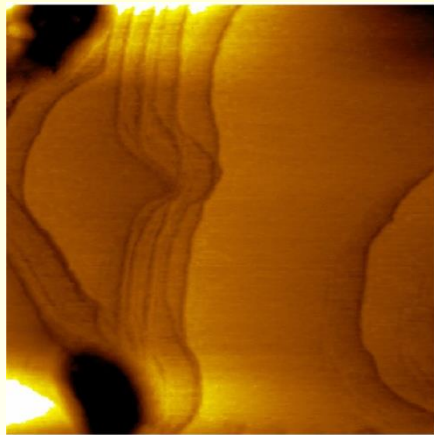


# Fast and slow processes

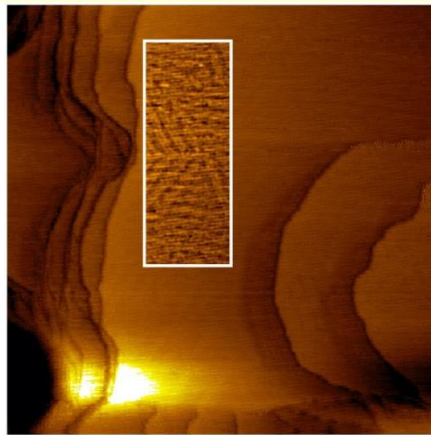


## Fast and slow processes

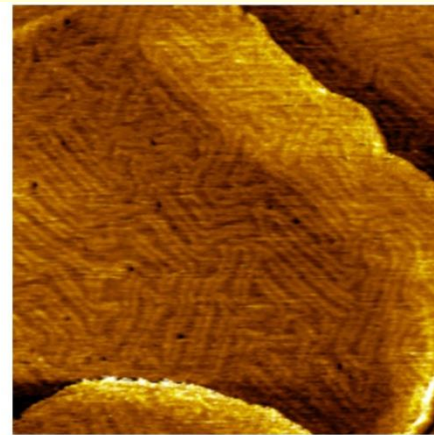
increasing negative potential / charge of the Au(111) electrode



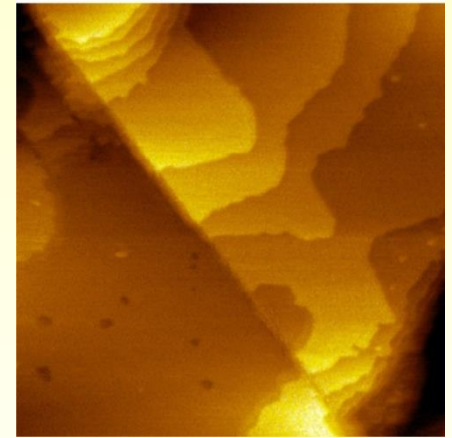
250 nm x 250 nm



250 nm x 250 nm



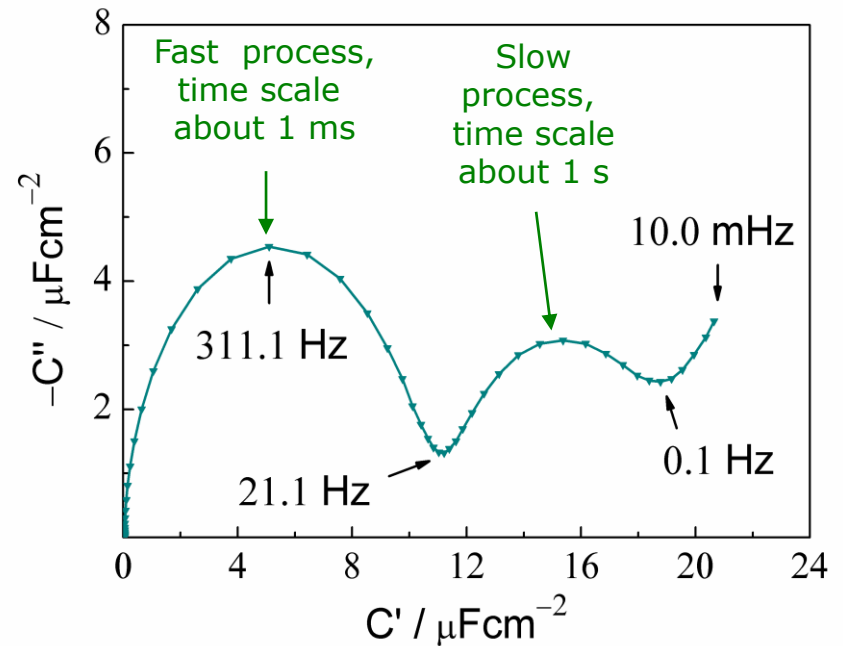
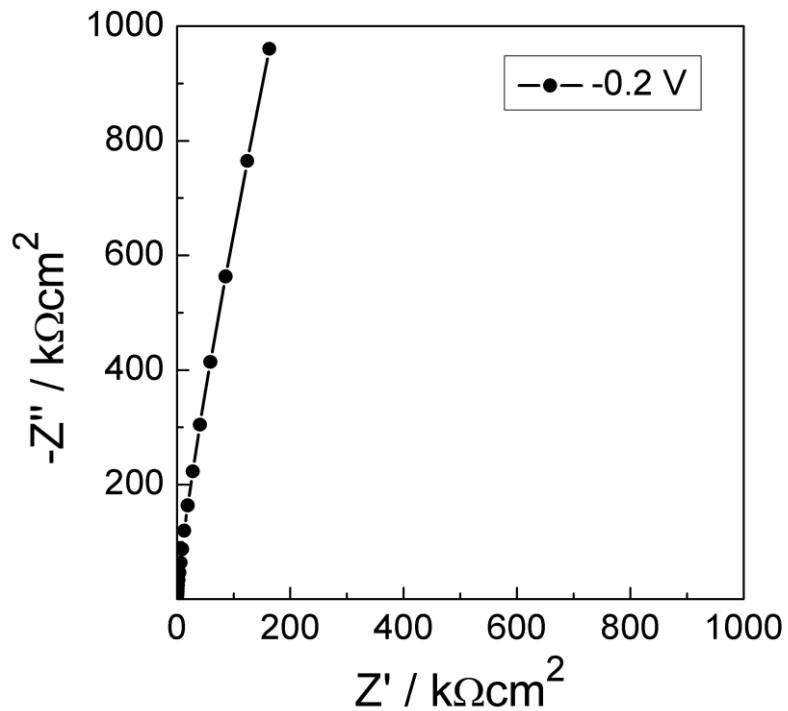
120 nm x 120 nm



Herringbone-type  
superstructure  
of Au surface

Maximum of  $\Delta C_{\text{slow}}$

# Impedance vs. capacitance spectra



**Two capacitive processes  
clearly visible**

Many thanks

for

your attention!