Nuclear Magnetic Resonance (NMR) – Some Basics and Comparison with BDS

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Outline

- Brief Introduction to NMR Spectroscopy
  - Interactions of Spins with External Fields
  - Interactions of Spins with Internal Fields

- NMR Studies of Molecular Dynamics
  - Spin-Lattice Relaxation
  - 1D and 2D NMR Spectra
  - Stimulated Echoes

Less than half a page, but worth half a Nobel Prize
Nobel Laureates in NMR Spectroscopy

**Bloch & Purcell (Physics 1952)**

… for their development of new methods for nuclear magnetic precision measurements

**Ernst (Chemistry 1991)**

… for his contributions to the development of the methodology of high-resolution NMR spectroscopy

**Wüthrich (Chemistry 2002)**

… for his development of NMR spectroscopy for determining the 3D structure of biological macromolecules in solution

**Lauterbur & Mansfield (Medicine 2003)**

… for their discoveries concerning magnetic resonance imaging
Some Basics Means: Many Topics not Covered

Chemical Solution NMR
Stuff with Fancy Acronyms
Spectral Editing, …

MQ-MAS Experiments
Magic Angle Spinning
Multiple-Quantum NMR, …

Double Resonance Techniques
Dipolar Decoupling
Cross Polarization
SEDOR, REDOR, …

Imaging
functional MRI, …

This tutorial focuses on correlation functions and molecular dynamics
Spins in Static Magnetic Fields

A nuclear spin $\vec{I}$ results in a nuclear magnetic moment $\mu$:

$$\vec{\mu} = \gamma (\hbar \vec{I})$$

$\gamma$: gyromagnetic ratio

Application of a magnetic field $\vec{B}_0 = (0,0,B_0)$:

Zeeman splitting into $2I + 1$ spin states

$$\mu_z = \gamma \hbar m$$

$m = I, I - 1, ..., -I$

$I$: spin quantum number
$m$: magnetic quantum number
Spins in Static Magnetic Fields – Energy

Example:  \( I = \frac{1}{2}, \gamma > 0 \) \((^1\text{H}, ^{13}\text{C}, ^{31}\text{P}, \ldots)\)

\[ E = \begin{align*}
E_m &= -\mu_z B_0 = -m \gamma \hbar B_0 \\
\Delta E &= \gamma \hbar B_0 = \hbar \omega_0
\end{align*} \]

Energy levels

Energy difference

Larmor frequency

Typical energy differences correspond to Larmor frequencies in the MHz regime (radio frequencies)
Spins in Static Magnetic Fields – Magnetization

\[ m = -\frac{1}{2} \]

\[ m = +\frac{1}{2} \]

\[ E_m = -\mu_z B_0 = -m \gamma \hbar B_0 \]

Boltzmann distribution:

Magnetization \( \hat{M}_0 = \sum_i \hat{\mu}_i \)

\[ \Delta E \ll k_B T: \quad \text{Curie law: } \hat{M}_0 = (C/T) \hat{B}_0 \quad C \propto N \gamma^2 \]

Signal-to-noise ratio depends on:

- gyromagnetic ratio \( \gamma \)
- magnetic field \( B_0 \)
- natural abundance \( (N) \)

Note: The population difference at 300 K in typical (few Tesla) fields is only 1 millionth. → NMR magnetization is VERY small.
### Nuclei for NMR Experiments

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Spin</th>
<th>Gyromagnetic ratio $(10^7 \text{rad T}^{-1} \text{s}^{-1})$</th>
<th>NMR frequency at 2.35 T (MHz)</th>
<th>Natural abundance (%)</th>
<th>Relative sensitivity $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1\text{H}$</td>
<td>1/2</td>
<td>26.752</td>
<td>100.000</td>
<td>99.985</td>
<td>1.00</td>
</tr>
<tr>
<td>$^2\text{H}$</td>
<td>1</td>
<td>4.107</td>
<td>15.351</td>
<td>0.015</td>
<td>$1.45 \times 10^{-6}$</td>
</tr>
<tr>
<td>$^3\text{He}$</td>
<td>1/2</td>
<td>-20.380</td>
<td>76.181</td>
<td>$1.4 \times 10^{-4}$</td>
<td>$5.75 \times 10^{-7}$</td>
</tr>
<tr>
<td>$^7\text{Li}$</td>
<td>3/2</td>
<td>10.398</td>
<td>38.866</td>
<td>92.58</td>
<td>0.272</td>
</tr>
<tr>
<td>$^{13}\text{C}$</td>
<td>1/2</td>
<td>6.728</td>
<td>25.145</td>
<td>1.108</td>
<td>$1.76 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{14}\text{N}$</td>
<td>1</td>
<td>1.934</td>
<td>7.228</td>
<td>99.630</td>
<td>$1.00 \times 10^{-3}$</td>
</tr>
<tr>
<td>$^{15}\text{N}$</td>
<td>1/2</td>
<td>-2.712</td>
<td>10.137</td>
<td>0.370</td>
<td>$3.86 \times 10^{-6}$</td>
</tr>
<tr>
<td>$^{17}\text{O}$</td>
<td>5/2</td>
<td>-3.628</td>
<td>13.562</td>
<td>0.037</td>
<td>$1.08 \times 10^{-5}$</td>
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<tr>
<td>$^{19}\text{F}$</td>
<td>1/2</td>
<td>25.181</td>
<td>94.094</td>
<td>100.000</td>
<td>0.834</td>
</tr>
<tr>
<td>$^{23}\text{Na}$</td>
<td>3/2</td>
<td>7.080</td>
<td>26.466</td>
<td>100.000</td>
<td>$9.27 \times 10^{-2}$</td>
</tr>
<tr>
<td>$^{31}\text{P}$</td>
<td>1/2</td>
<td>10.841</td>
<td>40.481</td>
<td>100.000</td>
<td>$6.65 \times 10^{-2}$</td>
</tr>
<tr>
<td>$^{39}\text{K}$</td>
<td>3/2</td>
<td>1.250</td>
<td>4.672</td>
<td>93.100</td>
<td>$4.75 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^{129}\text{Xe}$</td>
<td>1/2</td>
<td>-7.452</td>
<td>27.856</td>
<td>26.44</td>
<td>$5.71 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

$^a$ Relative sensitivity is the ratio of the sensitivity of the nucleus to that of hydrogen.

$^1\text{H}$, $^2\text{H}$, $^{19}\text{F}$, $^{31}\text{P}$, and … well suited for NMR experiments,

$^{12}\text{C}$ and $^{16}\text{O}$ do not possess a nuclear spin and $^{13}\text{C}$ and $^{17}\text{O}$ (and also $^2\text{H}$) studies usually require isotopic enrichment
Time Evolution of the Magnetization

If the magnetization is deflected from the equilibrium direction:

\[
\begin{align*}
M_x(t) &= M_{x,0} \cos(\omega_0 t) - M_{y,0} \sin(\omega_0 t) \\
M_y(t) &= M_{y,0} \cos(\omega_0 t) + M_{x,0} \sin(\omega_0 t) \\
M_z(t) &= M_{z,0}
\end{align*}
\]

Precession around the magnetic field \( \vec{B}_0 \):
\[
\omega_0 = \gamma B_0 \quad \text{(Larmor frequency)}
\]

\( \vec{M} \) is stationary in a coordinate system rotating with the Larmor frequency \( \omega_0 \) around \( \vec{B}_0 \) (rotating frame)

Classical Bloch equation:
\[
d\vec{M}/dt = \gamma (\vec{M} \times \vec{B}_0)
\]
Two perspectives

**static (lab-) frame**

**rotating frame**

\[ M_{\text{rot}}(t) = M_0 e^{i(\omega_L - \omega)t} \]
**Spins in Alternating Magnetic Fields**

**Irradiation with resonant radio-frequency pulses**

The sample resides in a coil, which belongs to a resonant circuit and enables application of an oscillating magnetic field $\vec{B}_1 \perp \vec{B}_0$:

$$\vec{B}_1(t) = 2\vec{B}_{1,0} \cos(\omega_0 t) \quad (\omega_{rf} = \omega_0)$$

In the rotating frame, the relevant of the two rotating components is static:

\[
d\hat{M}/dt = \gamma (\hat{M} \times \vec{B}_0)
\]

During application of $\vec{B}_1$:

Precession of $\hat{M}$ around $\vec{B}_1$ with a nutation frequency $\omega_1 = \gamma B_1$

$\vec{B}_1$ field is applied as radio-frequency pulse

90° pulse: $t_{90°} = \frac{\pi/2}{\omega_1} \approx \mu s$
The time-dependent magnetization in the laboratory system is detected via the voltage induced in the coil.
Various types of NMR experiments designed to study structure and dynamics on different time scales and length scales

- **Homogeneous fields**
- **Static samples**
- **Single resonance**

- **Gradient fields**
- **Rotating samples**
- **Multiple resonance**
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Spin Interactions in Solid-State NMR

Zeeman interaction

\[ \Delta E = \hbar \omega_0 = \hbar \gamma B_0 \]

Zeeman interaction + internal interaction

\[ \Delta E = \hbar (\omega_0 + \omega_{\text{int}}) \]

We observe a frequency shift \( \omega \)
(solid state: kHz)
Spin Interactions in Solid-State NMR

Dipolar interaction:
Interaction between the nuclear magnetic moments $\mu$ of the various nuclei

Chemical shift interaction:
Shielding of the magnetic field $B_0$ at the nucleus due to the electron clouds

Quadrupolar interaction ($I \geq 1$):
Interaction of the nuclear electric quadrupole moment with an electric field gradient at the nuclear site

The interaction energies depend on molecular orientations or molecular environments, i.e., on their interaction tensors

M. H. Levitt, Spin Dynamics
Example: Anisotropic Chemical Shielding (or Shift)

Different orientations of a C$_2$H$_2$ molecule relative to the $B_0$ field.

- Rotational symmetry: large diamagnetic ring current, high (chemical) shielding.
- No rotational symmetry: small diamagnetic ring current, little (chemical) shielding.

$^{13}$C NMR frequency $\omega$. 
Chemical Shift: Not only anisotropic but also *asymmetric*

Think of the electronic shielding cloud as of a squeezed american football

\[ \omega_{\text{aniso}}(\theta, \phi) = \omega_{\text{iso}} \pm \frac{\delta}{2} \left[ 3\cos^2 \theta - 1 - \eta \sin^2 \theta \cos (2\phi) \right] \]

*anisotropy* \( \delta = 0 \): for spherical tensor (soccer ball)

*asymmetry* \( \eta = 0 \): for axial tensor (non-squeezed american football)

http://anorganik.uni-tuebingen.de/klaus/nmr/index.php?p=conventions/csa/csa#converter
Powder Line Shapes

In crystalline powders and amorphous materials, a distribution of molecular orientations exists (powder average)

Due to the dependence of the frequency $\omega_{\text{aniso}}(\theta,\phi)$ on the orientation, *broad powder spectra* are observed

Maximum spectral width $\approx$ given by $\delta$

Shape by asymmetry of tensor, e.g., by $\eta$

Powder Line Shapes Containing Structural Information

CSA* interaction for $I = 1/2$

($^{31}$P NMR studies of phosphate glasses)

$$
\sigma_{\text{PAS}} = \begin{pmatrix}
\sigma_{xx} & 0 & 0 \\
0 & \sigma_{yy} & 0 \\
0 & 0 & \sigma_{zz}
\end{pmatrix}
$$

- **axially symmetric**
  - $\sigma_{xx} \neq \sigma_{yy} \neq \sigma_{zz}$
  - $\delta > 0, \eta > 0$

- **spherically symmetric**
  - $\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$
  - $\delta = 0, \eta = 0$

* Chemical Shift Anisotropy
Example 2: Quadrupolar Interaction for Spins \( I \geq 1 \)

Monitor bond orientations of molecular or polymeric systems using \(^2\text{H} \) NMR

Quadrupolar frequency (often \( \eta \approx 0 \))

\[
\omega_Q(\theta) = \pm \frac{\delta_Q}{2} (3\cos^2 \theta - 1)
\]

\( Q \): quadrupole moment of the nucleus
\( \Phi \): electric potential of the environment

\( I = 1 \) spins:
- two transitions
- two subspectra
- Pake pattern →

\( m = -1 \)
\( m = 0 \)
\( m = 1 \)
Comparison of NMR and BDS

**BDS:**
Electrical dipole moment vector
in electrical field

**NMR:**
chemical shift, quadrupolar, … tensor
in appropriate field

Potential energy

\[ W_{\text{diel}} = \vec{\mu} \vec{E} = \mu E \cos \theta = \mu E P_1(\cos \theta) \]

\[ W_{\text{NMR}} = \hbar \omega = \hbar \delta \frac{1}{2} (3 \cos^2 \theta - 1 - \eta \sin^2 \theta \cos 2\phi) \]

\[ = \hbar \delta P_2(\cos \theta) \quad \text{if } \uparrow \text{ negligible} \]

Higher rank \( l \) of
Legendre polynomial \( P_l(\cos \theta) \)
\rightarrow higher angular sensitivity

also \( l = 2 \): Kerr effect,
dynamic light scattering (DLS), ….
Measurement of Solid-State 1D NMR Spectra

The same coil is used for application of rf pulse and for signal detection. NMR signal cannot be observed immediately after the pulse.

For solids, echo-pulse sequences are used to overcome this “dead time”.

Hahn echo \((I = \frac{1}{2})\)

Fourier transformation of the signal after echo maximum yields undistorted spectrum.
Hahn's Echo and his Racetrack Analogy

E. Hahn, Free Nuclear Induction
*Physics Today* 6 (Nov. 1953) 4-9.

Cover image
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Deuteron NMR Studies of Molecular Dynamics

Effects of molecular dynamics in $^2$H NMR ($\eta = 0$)

The resonance frequency $\omega$ depends on the orientation of the C-D, N-D, or O-D bond:

$$\omega(\theta) = \pm \delta/2 \left( 3\cos^2 \theta - 1 \right) \propto P_2(\cos \theta)$$

Molecular reorientation renders $\omega$ time dependent

$$\omega_1(\theta_1) \quad \rightarrow \quad \omega_2(\theta_2)$$
Basics of Molecular Dynamics

Thermal motion is a stochastic process

Molecular orientations vary randomly in time: $f(\theta(t))$

Auto-Correlation Function

$$C(t) = \frac{\langle f(\theta(0))f(\theta(t)) \rangle}{\langle f(\theta(0))f(\theta(0)) \rangle}$$

Fourier-transformation

Spectral Density

$$J(\omega) \propto \text{Re} \left[ \int_0^\infty C(t) \exp(-i\omega t) dt \right]$$
$^2$H NMR Spin-Lattice Relaxation

Strong fluctuations of the quadrupolar frequency: $\omega_Q(\theta) \propto P_2(\cos \theta)$
on the scale of the Larmor frequency
leads to fast spin-lattice relaxation

Relation between spin relaxation rate und spectral density

**Bloembergen-Purcell-Pound (BPP)**

$$\frac{1}{T_1} = \frac{2}{15} \delta^2 \cdot [J(\omega_0) + 4J(2\omega_0)]$$

Correlation function of rank $l = 2$
Legendre polynomial: $f(\theta) = P_2(\cos \theta)$

$$C_2(t) = \frac{\langle f(\theta(0)) f(\theta(t)) \rangle}{\langle f(\theta(0)) f(\theta(0)) \rangle}$$

Spectral density

$$J(\omega) = \int_0^\infty C_2(t) \cos(\omega t) dt$$

$$= \text{Re} \left[ \int_0^\infty C_2(t) e^{-i\omega t} dt \right]$$

second term not important, e.g., for CSA interaction
Example: Exponential Correlation Function

\[ C_2(t) = \exp\left(-\frac{t}{\tau}\right) \]

\[ J(\omega) = \frac{\tau}{1 + (\omega \tau)^2} \]

\[ \frac{1}{T_1} \propto \frac{\tau}{1 + (\omega \tau)^2} + \ldots \]

NMR: Spectral density measure for fluctuation probability.
Normalization: \( \int J(\omega) \, d\omega = 1 \)

\( T_1 \) minimum at \( \omega_0 \tau \approx 0.62 \approx 1 \)

Simple relation: Relaxation time \( T_1 \) vs. correlation time \( \tau \)
Spin-Lattice Relaxation and Dielectric Loss

Multiplication with frequency yields expression like loss (= dissipation) component of Debye relaxator

\[
\frac{\omega}{T_1} \propto \frac{\omega \tau}{1 + (\omega \tau)^2} + \ldots
\]

(classical) linear response theory
→ Fluctuation-Dissipation-Theorem (FDT)

\[
J(\omega) = \frac{k_B T}{\pi \omega} \varepsilon''(\omega)
\]

often a good approximation if:
- \(J(2\omega)\) term neglected
- difference in rank \(l\) neglected
- dielectric cross terms neglected

A \(1/T_1\) measurement at a single (Larmor) frequency corresponds (roughly) to a single-frequency measurement of the dielectric loss

NMR measurements at different Larmor frequencies are possible
Two Experimental Examples

red: from dielectric response: 
**dissipation**

black: from NMR spectral density: 
**fluctuation**

site specificity by isotope labeling of (here:) butanol

Blochowicz et al., JCP 110, 12011 (1999)
Gainaru et al., PRL 105, 258303 (2010)
Field Cycling: Broadband NMR Relaxometry

\[ \frac{1}{T_1} \rightarrow \nu / T_1 \propto \chi'' \]

susceptibility representation

frequency scaling

OTP: clean low-frequency flank

Many applications of FC-NMR also for polymers
Measuring Nuclear Spin-Lattice Relaxation

(Boltzmann) equilibrium of spin levels
invert magnetization by 180° pulse

"lattice" = environment of the nucleus, no matter whether crystalline, amorphous, or liquid

\[ M(t) = M_0 + \left[ M(0) - M_0 \right] e^{-t/T_1} \]
Other Ways to Access Spectral Densities I: Dielectric Noise

lines: conventional dielectric measurements

points: dedicated voltage noise experiments

\[ S_V = 4k_B T \text{ Re}[Z(\nu)] \]

Nyquist theorem

analogously used in AFM approaches
e.g., Russell, Israeloff, Nature 408, 695 (2000); picture: Hoepker et al., JPCB 115, 1449 (2011)

see also: Schildmann, Reiser, Gainaru, Gainaru, Böhmer, JCP 135, 174511 (2011)
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Excellent book for 2D, 3D, … NMR!

Impossible to cover in 50 minutes → Just one slide with spectra!
**Effect of Molecular Dynamics on NMR Line Shapes**

Fast isotropic reorientation: \( \tau < 1/\delta \approx 1 \mu s \)

Spectra useful to determine rate and jump geometry of rotational motions.

Beautiful 2D spectra result if well-defined large-angle (> 30°) jumps dominate the reorientation.

Example:

\(^2\)H-NMR of glycerol-d\(_5\)

No motion on a time scale of \(1/\delta \approx 1 \mu s\)

Tetrahedral jumps of benzene-d\(_6\) in Na-Y zeolite, Geil et al., JCP 116, 2184 (2002)

Vogel, Rössler, JPCB 104, 4285 (2000)
Can We Measure Correlation Functions Directly?

Apply Broadband Neutron Spectroscopy → density-density ($\rho$-$\rho$) correlation for time difference $t = t_2 - t_1$

$$S_{\text{inc}}(\vec{Q}, t) = \langle \rho(\vec{Q}, t_1) \rho(\vec{Q}, t_2) \rangle$$

Intermediate scattering function expressed as phase correlation function:

$$S_{\text{inc}}(\vec{Q}, t) = \left\langle \exp[+i\vec{Q}\vec{r}(t_1)] \exp[-i\vec{Q}\vec{r}(t_2)] \right\rangle$$ sensitive to translational coordinate $\vec{r}$

Consider rotational analog: Replace $\vec{r}$ by 'rotational coordinate' $\vec{\omega}(\theta)$

$$S_{\text{rot}}(\vec{Q}_{\text{rot}}, t) = \left\langle \exp[+i\vec{Q}_{\text{rot}} \omega(t_1)] \exp[-i\vec{Q}_{\text{rot}} \omega(t_2)] \right\rangle$$

Here $\omega$ is the NMR frequency. What could $Q_{\text{rot}}$ be?
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Idea of the stimulated echo: A break in Hahn's race
Stimulated-Echo Experiment: Basic Sequence also for 2D NMR

Stimulated-echo sequence

\[ \omega_1 \] \hspace{1cm} \omega_2 \]
\[ t_1 \quad t_m \quad t_2 \]

"cut the refocusing pulse of the Hahn-echo sequence in two halves"

Measure and correlate NMR frequencies at two times separated by

\[ t_m \] is called mixing time

\[
S_{\text{rot}}(Q_{\text{rot}}, t) = \langle \exp[+iQ_{\text{rot}}\omega(t_1)] \exp[-iQ_{\text{rot}}\omega(t_2)] \rangle
\]

\[ Q_{\text{rot}} \] is called evolution time = \[ t_e \]

Terminology in NMR where only real (not: complex) quantities are measured

\[ \omega_{1,2} = \omega(t_{1,2}) \]

\[
F_2^{\text{cos}}(t_e, t_m) = \langle \cos(\omega_1 t_e) \cdot \cos(\omega_2 t_e) \rangle
\]

\[
F_2^{\text{sin}}(t_e, t_m) = \langle \sin(\omega_1 t_e) \cdot \sin(\omega_2 t_e) \rangle
\]

measure echo height as a function of \[ t_m \]!
**Rotational Correlation Functions:** \( \langle P_l(0)P_l(t) \rangle \)

m-tricresyl phosphate \((T_g = 214 \text{ K})\)

2-methyltetrahydrofuran \((T_g = 92 \text{ K})\)

\(31^P\) NMR stimulated-echo decays

\[
F_2^{\sin}(t_e, t_m) = \langle \sin(\omega_1 t_e) \cdot \sin(\omega_2 t_e) \rangle
\]

\[ t_e \rightarrow 0 \quad \Rightarrow \quad t_e^2 \langle \omega_1 \cdot \omega_2 \rangle \]

\[
\langle P_2[\cos \theta(0)]P_2[\cos \theta(t)] \rangle \rightarrow l = 2
\]

\[
\langle P_2[\cos \theta(0)]P_2[\cos \theta(t)] \rangle \rightarrow l = 1
\]

Rössler, Eiermann, JCP **100**, 5237 (1994)

Qi et al., JCP **118**, 7431 (2003)